

### Rendszerező, azonosító információk

<b>A fejlesztés címe</b>	Matematika felzárkóztató
<b>Azonosító</b>	T5.1
<b>A fejlesztésért felelős szervezet</b>	Matematikai és Statisztikai Modellezés Intézet / Matematika Tanszék
<b>A fejlesztésben résztvevők</b>	Tarlós Péter, Palágyi Zoltán
<b>A fejlesztés eredményének elérhetősége</b>	BCE, Tanárképző és Digitális Tanulás Központ honlap, Kutatás lap alatt
<b>Kapcsolattartó a további információkhoz</b>	Dr. Daruka Magdolna magdolna.daruka@uni- corvinus.hu

### A fejlesztés tartalma

### Tantárgyi adatlap

**EFOP-3.4.3-16-2016-00006**  
**„A Budapesti Corvinus Egyetem intézményi fejlesztései a felsőfokú oktatás minőségének és hozzáférhetőségének együttes javítása érdekében”**

The screenshot shows a Moodle course page for 'Matematika 3.4.3'. The browser address bar shows 'moodle.uni-corvinus.hu/course/view.php?id=13'. The page has a yellow header with 'Üzenetek' and user information 'magyar (hu) Csillik Olga'. The main content area is titled 'Matematika 3.4.3' and includes a 'Folyamatjelző' icon. Below this, there are four topic sections: 'Téma 1', 'Téma 2', 'Téma 3', and 'Téma 4'. Each topic contains a list of resources with checkboxes for completion. 'Téma 1' includes 'Mat Tallos Vektorterek alterek' (916.3KB PDF), 'aktív teszt', 'Gondolattérkép', 'Féléves feladatok', and 'h5p'. 'Téma 2' includes 'Mat Tallos Linearis fuggetlenség' (921.6KB PDF). 'Téma 3' includes 'Mat Tallos Linearis lekepezések' (891.7KB PDF). 'Téma 4' is currently empty. On the right side, there are two sidebars: 'Kurszus menü' and 'Adminisztráció'. The 'Adminisztráció' sidebar lists various course management options like 'Beállítások szerkesztése', 'Szerkesztés bekapcsolása', 'Kurszus teljesítése', 'Felhasználó', 'Jelentések', 'Osztályozónapló beállítása', 'Kitűzők', 'Importálás', and 'Kérdésbank'. Below it is the 'Felhasználói könyvjelzők' sidebar with the option 'Oldal felvétele a könyvjelzők közé'. At the bottom, a search bar contains 'matema' and shows 'Összes kijelölése', 'Kjs- és nagybetűk', 'Egész szavak', and '1 / 2 találat'. The Windows taskbar at the bottom shows the time as 11:25 on 2020.02.03.

**EFOP-3.4.3-16-2016-00006**  
**„A Budapesti Corvinus Egyetem intézményi fejlesztései a felsőfokú oktatás minőségének és hozzáférhetőségének együttes javítása érdekében”**

The screenshot shows a web browser window displaying a Moodle course page. The browser's address bar shows the URL `moodle.uni-corvinus.hu/course/view.php?id=13`. The page title is "Kurzus: Matematika 3.4.3". The user is logged in as "Csillik Olga" in Hungarian. The course navigation menu includes "Kezdőoldal", "Félévek", "Kursusaim", "Aktuális kurzus", "Szerkesztés bekapcsolása", "Blokkok", "Szélesség", and "Fejléc". The main content area lists several topics, each with a PDF document:

- Téma 4**: Mat Tallos Linearis egyenletrendszerek 961.5KB PDF-dokumentum
- Téma 5**: Mat Tallos Sajatertek sajátvektor 963.6KB PDF-dokumentum
- Téma 6**: Mat Tallos Determináns 909.7KB PDF-dokumentum
- Téma 7**: Mat Tallos Ketváltozos függvények 911KB PDF-dokumentum
- Téma 8**: Mat Tallos Felteteles szelsoertek 820.6KB PDF-dokumentum
- Téma 9**: (Empty)

The search bar at the bottom of the page contains the word "matema" and shows "Összes kijelölése", "Kjs- és nagybetűk", "Egész szavak", and "1 / 2 találat". The Windows taskbar at the bottom shows the time as 11:25 on 2020.02.03.

- Valószínűségszámítás
  - Bayes

52 lap, színiek közé.  
A = egyik lap elvétele.  
Húzott lapot, az az ásu lett. = A  
 $P(\text{ásu vettél el}) = ?$

$$P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A|B) \cdot P(B) + P(A|\bar{B}) \cdot P(\bar{B})} = \frac{\frac{3}{51} \cdot \frac{4}{52}}{\frac{3}{51} \cdot \frac{4}{52} + \frac{4}{51} \cdot \frac{48}{52}}$$

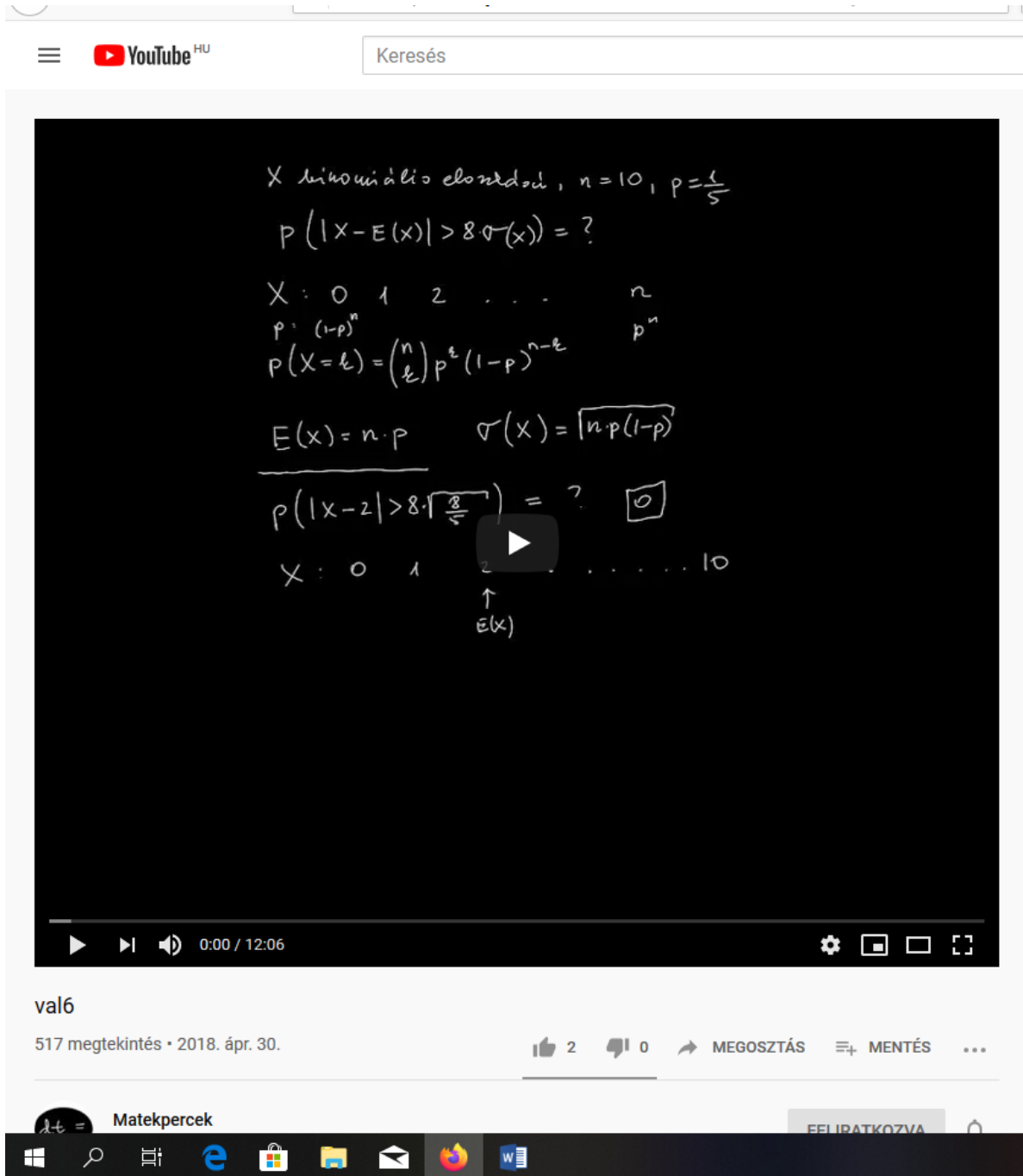
$$= \frac{3 \cdot 4}{3 \cdot 4 + 4 \cdot 48} = \frac{1}{17}$$

$$P(B) = \frac{4}{52} = \frac{1}{13}$$

val12  
441 megtekintés • 2018. máj. 6.    3    0    MEGOSZTÁS    MENTÉS

Matekpercek  
[https://www.youtube.com/watch?v=2gNSZgBLr\\_A](https://www.youtube.com/watch?v=2gNSZgBLr_A)    FELIRATKOZVA

Binomiális eloszlás



The video content shows the following handwritten text on a blackboard:

$X$  binomiális eloszlású,  $n=10$ ,  $p=\frac{1}{5}$

$P(|X - E(X)| > 8\sigma(X)) = ?$

$X: 0 \quad 1 \quad 2 \quad \dots \quad n$

$P: (1-p)^n \quad \dots \quad p^n$

$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$

$E(X) = n \cdot p \quad \sigma(X) = \sqrt{n \cdot p \cdot (1-p)}$

$P(|X - 2| > 8 \cdot \sqrt{\frac{8}{5}}) = ? \quad \square$

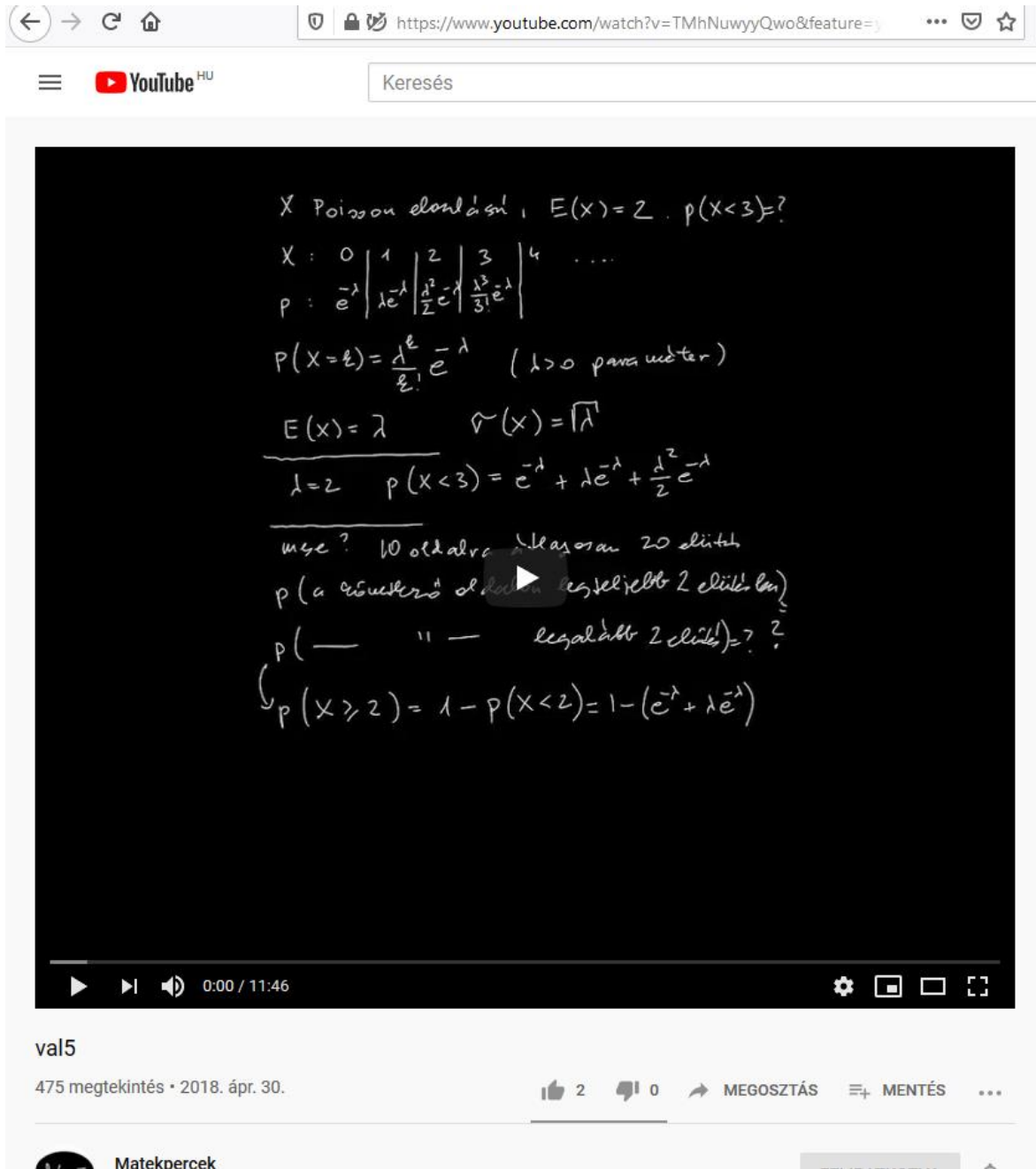
$X: 0 \quad 1 \quad 2 \quad \dots \quad 10$

↑  
 $E(X)$

Below the video player, the YouTube interface shows the video title "val6", 517 views, and a date of 2018. ápr. 30. The video player controls at the bottom indicate a duration of 0:00 / 12:06.

Poisson eloszlás

példa1

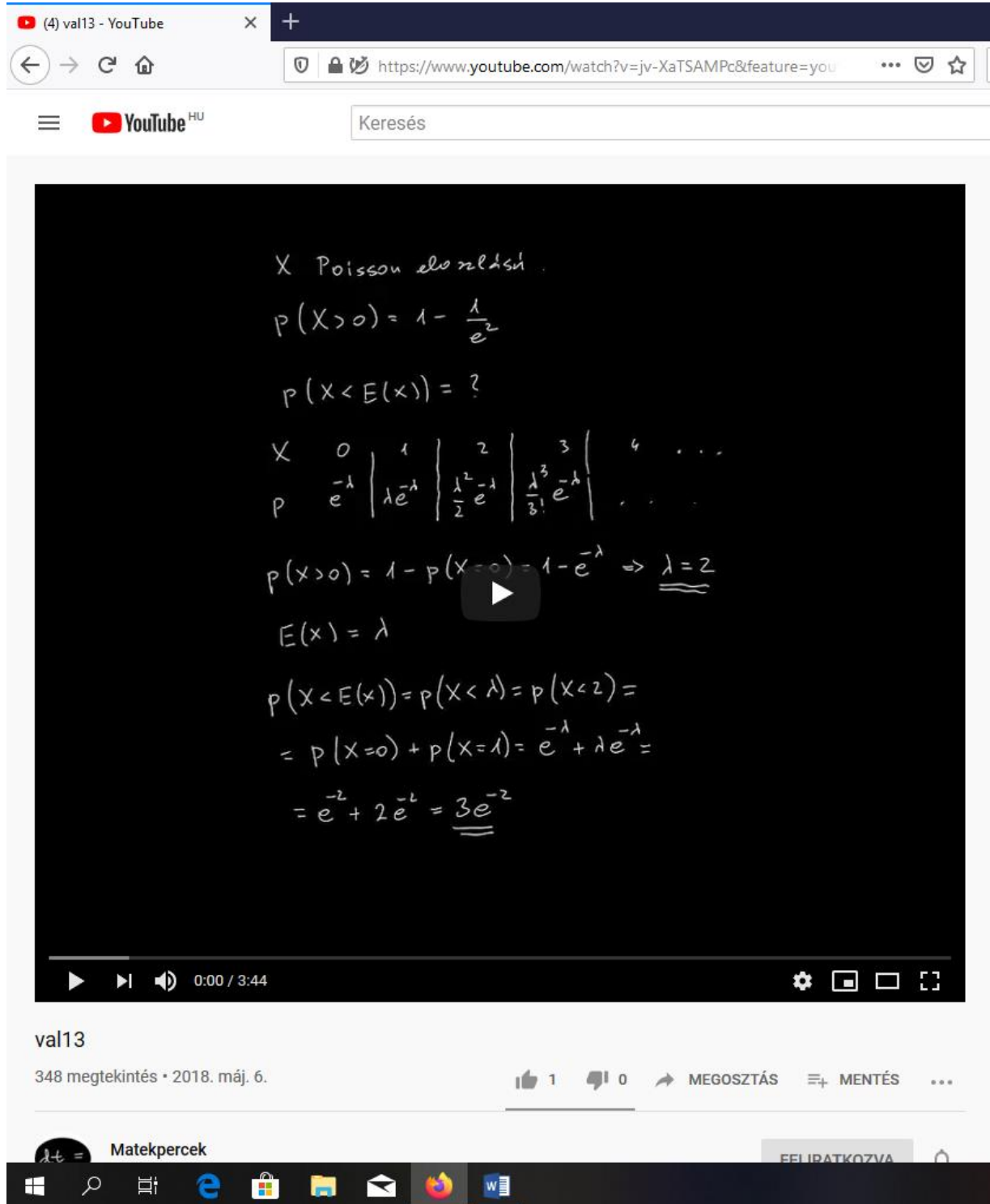


$X$  Poisson eloszlású,  $E(X) = 2$ .  $p(X < 3) = ?$   
 $X: 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid \dots$   
 $p: e^{-\lambda} \mid \lambda e^{-\lambda} \mid \frac{\lambda^2}{2} e^{-\lambda} \mid \frac{\lambda^3}{3!} e^{-\lambda} \mid \dots$   
 $P(X=k) = \frac{\lambda^k}{k!} e^{-\lambda}$  ( $\lambda > 0$  paraméter)  
 $E(X) = \lambda$       $\sigma(X) = \sqrt{\lambda}$   
 $\lambda = 2$       $p(X < 3) = e^{-2} + 2e^{-2} + \frac{2^2}{2} e^{-2}$   
 msc? 10 oldalra választva 20 elütés  
 $p(\text{a esemény 10 oldalra választva legfeljebb 2 elütésben})$   
 $p(\text{--- " --- legalább 2 elütés}) = ?$   
 $p(X \geq 2) = 1 - p(X < 2) = 1 - (e^{-\lambda} + \lambda e^{-\lambda})$

val5  
475 megtekintés • 2018. ápr. 30.     2     0     MEGOSZTÁS     MENTÉS     ...

Matekpercek

példa2



The screenshot shows a YouTube video player with a handwritten mathematical solution for a Poisson distribution problem. The text in the video is as follows:

X Poisson eloszlás.

$$P(X > 0) = 1 - \frac{1}{e^2}$$

$$P(X < E(X)) = ?$$

X	0	1	2	3	4	...
P	$e^{-\lambda}$	$\lambda e^{-\lambda}$	$\frac{\lambda^2}{2} e^{-\lambda}$	$\frac{\lambda^3}{3!} e^{-\lambda}$	...	...

$$P(X > 0) = 1 - P(X = 0) = 1 - e^{-\lambda} \Rightarrow \underline{\underline{\lambda = 2}}$$

$$E(X) = \lambda$$

$$P(X < E(X)) = P(X < \lambda) = P(X < 2) =$$

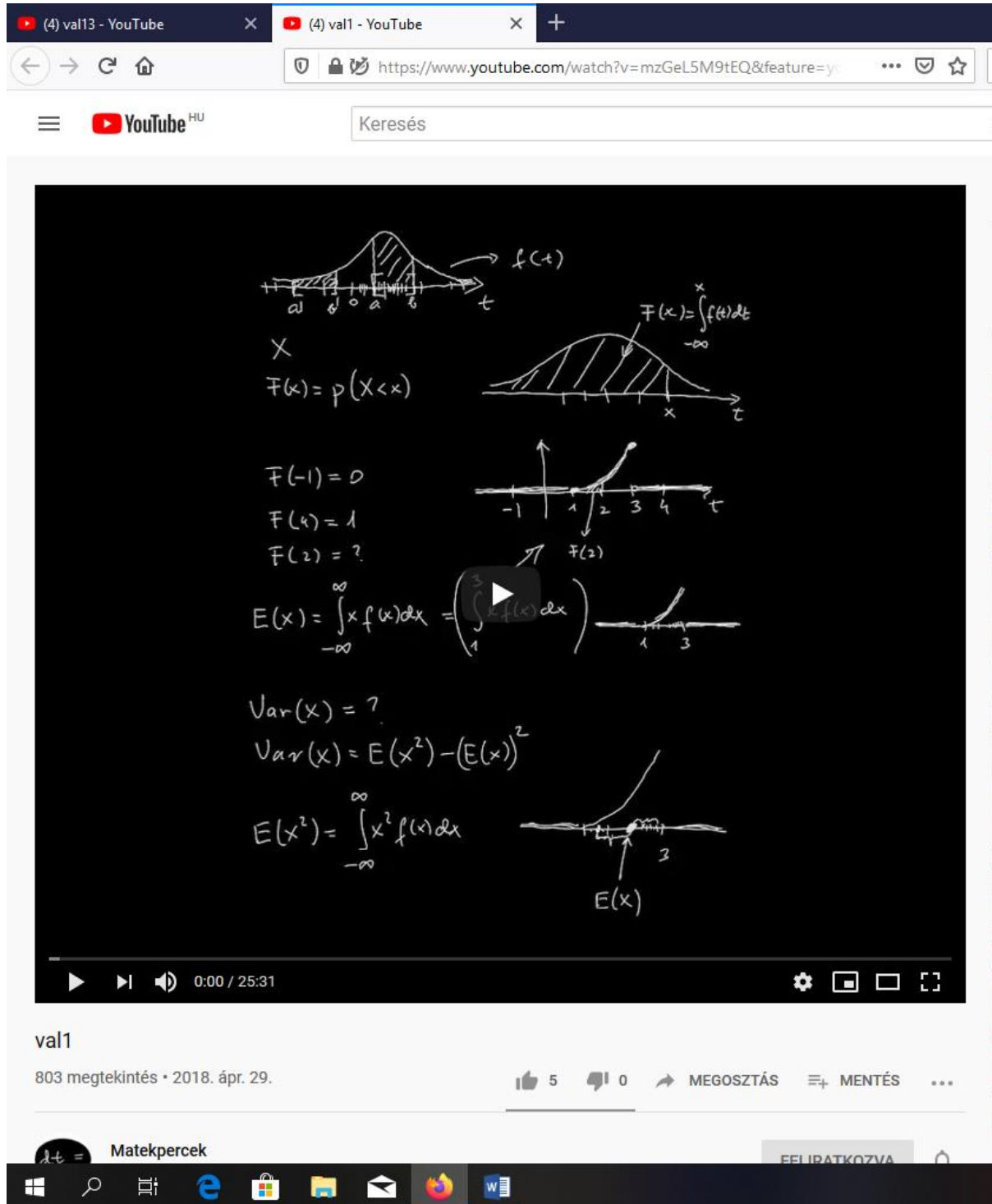
$$= P(X = 0) + P(X = 1) = e^{-\lambda} + \lambda e^{-\lambda} =$$

$$= e^{-2} + 2e^{-2} = \underline{\underline{3e^{-2}}}$$

Below the video player, the video title is "val13", it has 348 views, and was uploaded on May 6, 2018. The channel name is "Matekpercek".

Sűrűségfüggvény, eloszlásfüggvény, várható érték, szórás

Bevezető:



The screenshot shows a YouTube video player with handwritten mathematical content on a black background. The content includes:

- A graph of a probability density function  $f(t)$  with a shaded area under the curve between points  $a$  and  $b$ .
- The definition of the cumulative distribution function:  $F(x) = p(X < x)$ .
- The integral definition of the CDF:  $F(x) = \int_{-\infty}^x f(t) dt$ .
- Graphs illustrating  $F(-1) = 0$ ,  $F(4) = 1$ , and  $F(2) = ?$ .
- The formula for the expected value:  $E(x) = \int_{-\infty}^{\infty} x f(x) dx = \left( \int_1^3 x f(x) dx \right)$ .
- The formula for variance:  $Var(x) = E(x^2) - (E(x))^2$ .
- The formula for the second moment:  $E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$ .

Below the video player, the video title is "val1", it has 803 views, and was uploaded on 2018. ápr. 29. The channel name is "Matekpercek".



Példa:



$f(t) = \begin{cases} 4t^3, & \text{ha } 0 < t \leq A \\ 0 & \text{egyébként} \end{cases}$

$A = ? \quad F(x) = ? \quad E(x) = ?$

$\int_0^A 4t^3 dt = \left[ t^4 \right]_0^A = A^4 - 0^4 = A^4 = 1 \quad \boxed{A=1}$

$F(x) = \begin{cases} 0 & x \leq 0 \\ x^4 & 0 < x < 1 \\ 1 & x \geq 1 \end{cases} \quad \int_0^x 4t^3 dt = \left[ t^4 \right]_0^x = x^4$

$P(X > \frac{1}{2}) = 1 - F(\frac{1}{2}) = 1 - (\frac{1}{2})^4$

$E(x) = \int_{-\infty}^{\infty} x f(x) dx = \int_0^1 x \cdot 4x^3 dx = 4 \int_0^1 x^4 dx = 4 \cdot \left[ \frac{x^5}{5} \right]_0^1 = \frac{4}{5}$

$Var(x) = E(x^2) - (E(x))^2 = E(x^2) - (\frac{4}{5})^2 = \left( \frac{2}{3} - (\frac{4}{5})^2 \right)$

$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx = \int_0^1 x^2 \cdot 4x^3 dx = 4 \int_0^1 x^5 dx = 4 \cdot \left[ \frac{x^6}{6} \right]_0^1 = \frac{4}{6} = \frac{2}{3}$

Szép munkát !!!

Példa (nincs szórás)

X valószínűségi változó sűrűség függvénye

$$f(t) = \begin{cases} \frac{A}{t^3}, & \text{ha } t > 1 \\ 0 & \text{egyébként} \end{cases}$$

$A = ?$   $F(x) = ?$   $E(x) = ?$   $V(x) = ?$

$$\int_1^{\infty} \frac{A}{t^3} dt = A \int_1^{\infty} \frac{1}{t^3} dt = A \left[ \frac{t^{-2}}{-2} \right]_1^{\infty} = A \left( 0 - \frac{1}{-2} \right) = A \cdot \frac{1}{2} = 1 \Rightarrow \underline{A=2}$$

$$F(x) = \begin{cases} 0, & \text{ha } x \leq 1 \\ 1 - \frac{1}{x^2}, & \text{ha } x > 1 \end{cases}$$

$$\int_1^x \frac{2}{t^3} dt = 2 \left[ \frac{t^{-2}}{-2} \right]_1^x = - \left[ \frac{1}{t^2} \right]_1^x = - \left( \frac{1}{x^2} - 1 \right) = 1 - \frac{1}{x^2}$$

$P(X < 2) = ?$

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx = \int_1^{\infty} x \cdot \frac{2}{x^3} dx = 2 \int_1^{\infty} \frac{1}{x^2} dx = 2 \left[ -\frac{1}{x} \right]_1^{\infty} = 2(0 - (-1)) = \underline{2}$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx = \int_1^{\infty} x^2 \cdot \frac{2}{x^3} dx = 2 \left[ \ln x \right]_1^{\infty} \quad \lim_{x \rightarrow \infty} \ln x = \infty$$

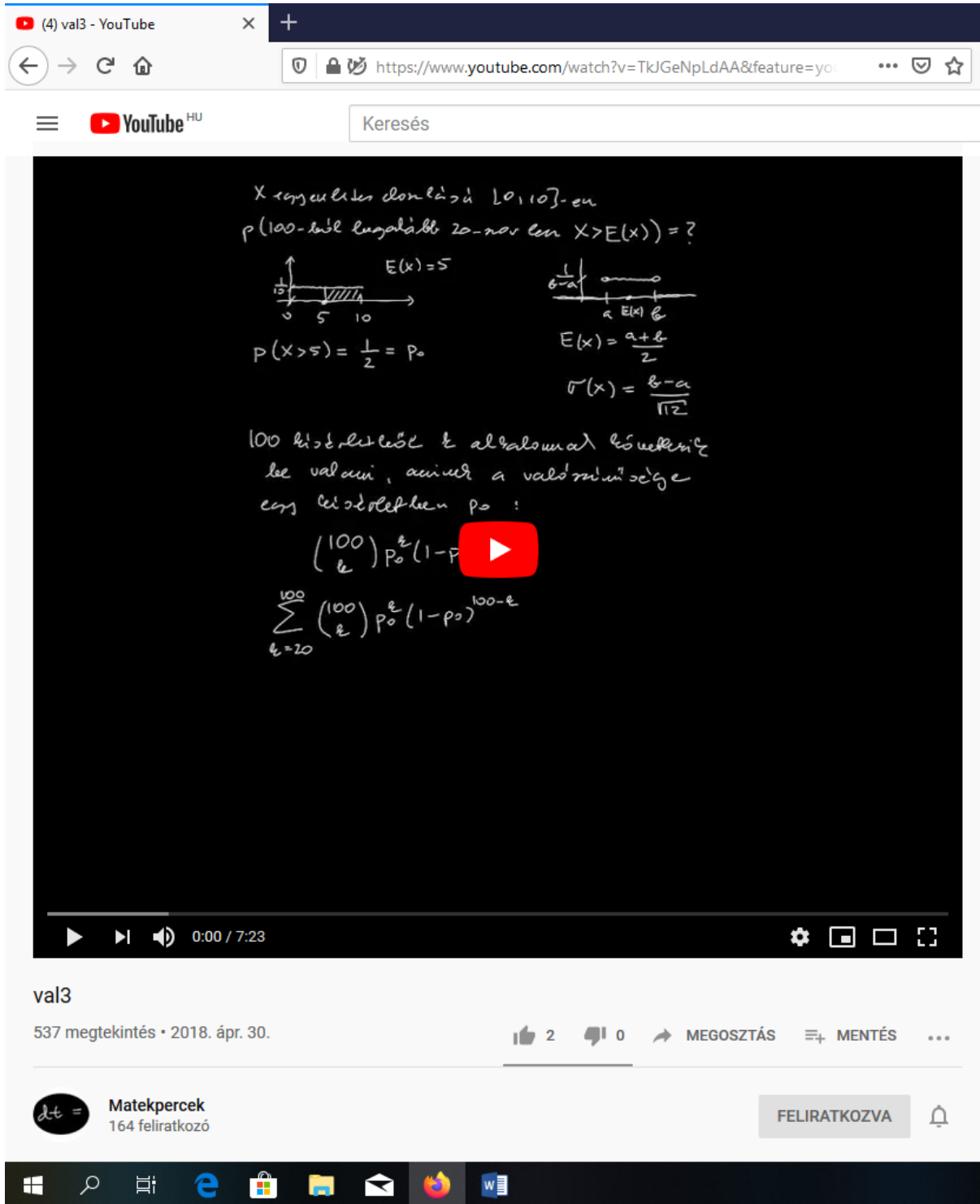
$(V(x) = \sqrt{E(x^2) - E(x)^2})$   $V(x)$  nem létezik!

val14  
478 megtekintés • 2018. máj. 6.

Matekpercek

Egyenletes eloszlás

Binomiálissal kombinálva



The video content includes the following text and formulas:

$X$  egyenletes eloszlású  $[0, 10]$ -en  
 $P(100$ -ból legalább  $20$ -at  $X > E(X)) = ?$

$E(X) = 5$   
 $P(X > 5) = \frac{1}{2} = p_0$

$E(X) = \frac{a+b}{2}$   
 $\sigma(X) = \frac{b-a}{\sqrt{12}}$

100 kísérletből  $k$  alkalommal előfordul  
 az  $X > 5$ , amihez a valószínűség  
 egy kísérletben  $p_0$ :

$$\binom{100}{k} p_0^k (1-p_0)^{100-k}$$

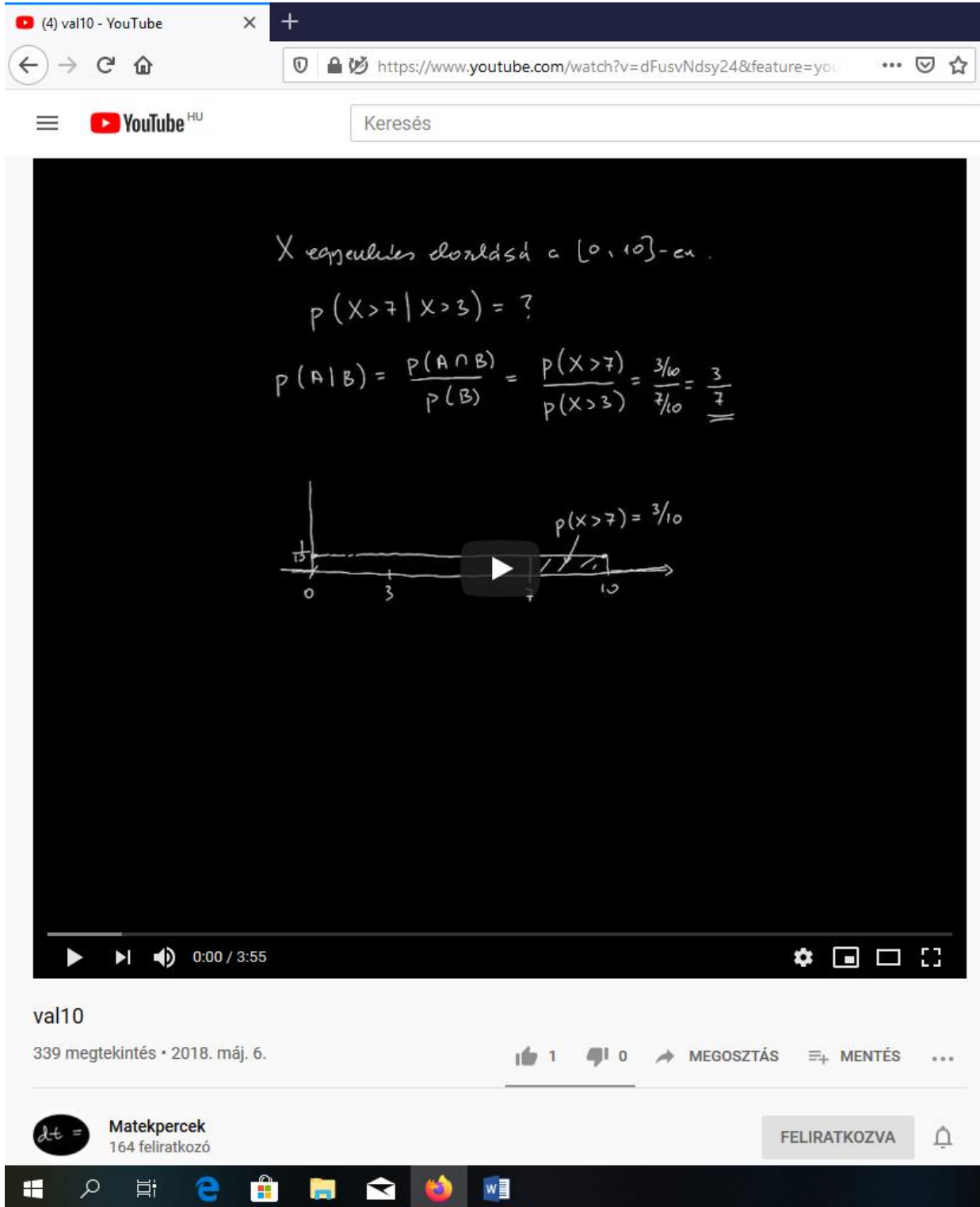
$$\sum_{k=20}^{100} \binom{100}{k} p_0^k (1-p_0)^{100-k}$$

Video player controls: 0:00 / 7:23

Channel: Matekpercek (164 feliratkozó)

Engagement: 537 megtekintés • 2018. ápr. 30. (2 likes, 0 comments)

## Feltételes valószínűség



The screenshot shows a YouTube video player with a handwritten solution to a conditional probability problem. The text in the video is as follows:

$X$  egyenletes eloszlású a  $[0, 10]$ -en.

$P(X > 7 | X > 3) = ?$

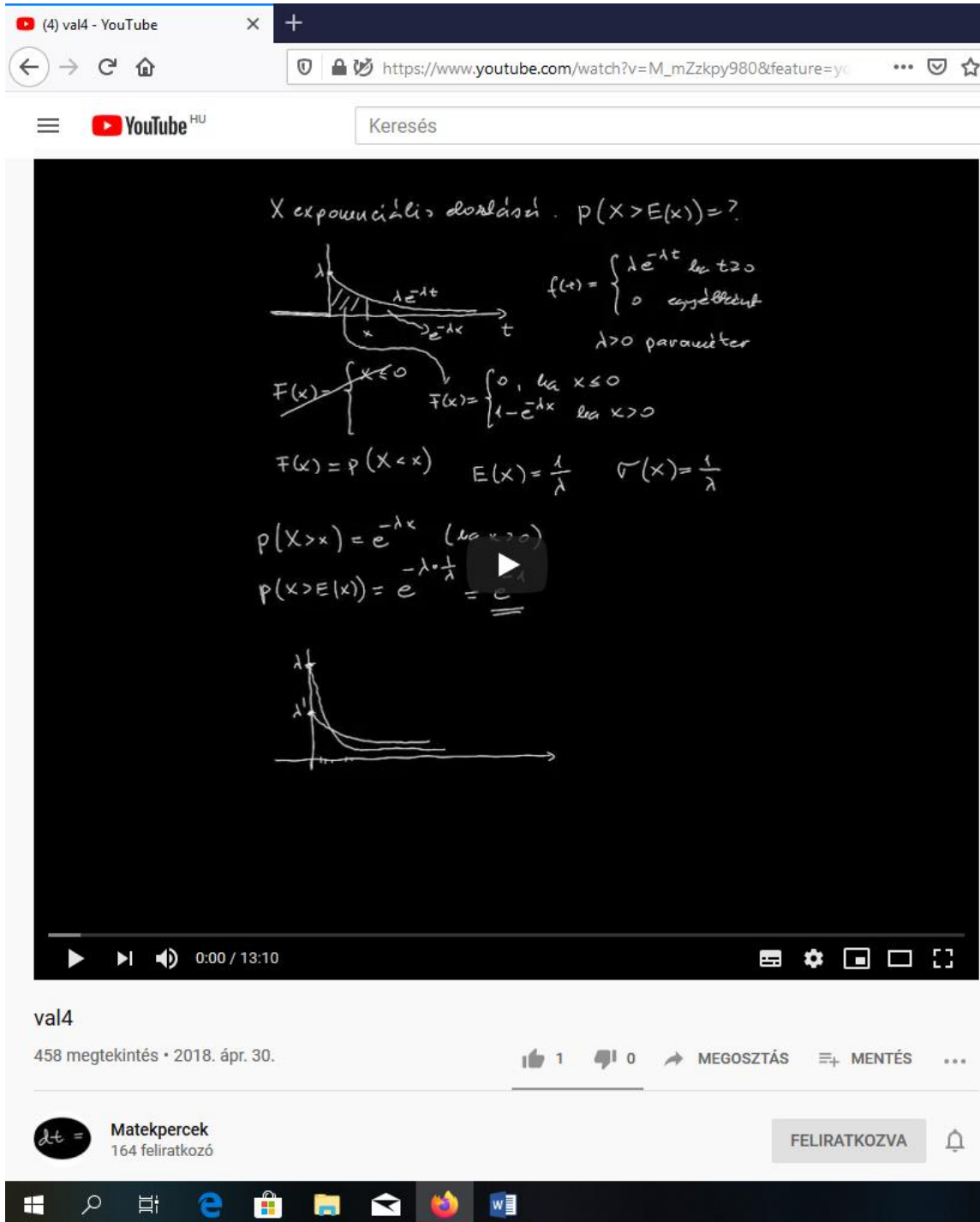
$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(X > 7)}{P(X > 3)} = \frac{3/10}{7/10} = \underline{\underline{\frac{3}{7}}}$$

Below the text is a number line from 0 to 10. A vertical line is drawn at 3, and another at 7. The region between 3 and 10 is shaded, representing the event  $X > 3$ . Within this shaded region, the sub-region between 7 and 10 is also shaded, representing the event  $X > 7$ . A label  $P(X > 7) = 3/10$  points to the region between 7 and 10.

The video player interface shows the video title "val10", 339 views, and the channel name "Matekpercek" with 164 subscribers. The video is set to "FELIRATKOZVA" (Subscribed).

## Exponenciális eloszlás

### Példa



(4) val4 - YouTube

https://www.youtube.com/watch?v=M\_mZzkpy980&feature=y...

Keresés

X exponenciális eloszlású.  $p(X > E(X)) = ?$

$f(t) = \begin{cases} \lambda e^{-\lambda t} & \text{ha } t \geq 0 \\ 0 & \text{egyébként} \end{cases}$   
 $\lambda > 0$  paraméter

$F(x) = \begin{cases} 0 & \text{ha } x \leq 0 \\ 1 - e^{-\lambda x} & \text{ha } x > 0 \end{cases}$

$F(x) = P(X < x)$      $E(X) = \frac{1}{\lambda}$      $\sigma(X) = \frac{1}{\lambda}$

$P(X > x) = e^{-\lambda x}$  (ha  $x > 0$ )  
 $P(X > E(X)) = e^{-\lambda \cdot \frac{1}{\lambda}} = e^{-1} = \frac{1}{e}$

0:00 / 13:10

val4

458 megtekintés • 2018. ápr. 30.

1 0 MEGOSZTÁS MENTÉS

Matekpercek  
164 feliratkozó

FELIRATKOZVA

Binomiálissal kombinálva



The video content includes the following text and equations:

$X$  exponenciális eloszlású,  $\lambda = 5$ .

$P(100 - \text{bél legkelembb 20 alkalommal len } X > 1) = ?$

$P(X > 1) = ?$

$P(X > 1) = e^{-\lambda \cdot 1} = e^{-5}$

$p_0 = e^{-5}$

$P = \sum_{k=0}^{20} \binom{100}{k} p_0^k (1-p_0)^{100-k}$

The video also features a graph of the exponential distribution function  $P(X > x) = e^{-\lambda x}$  with a vertical line at  $x=1$  and a shaded area under the curve to the right of  $x=1$ . A note says "X-re egy megfigyelés".

Video title: val8

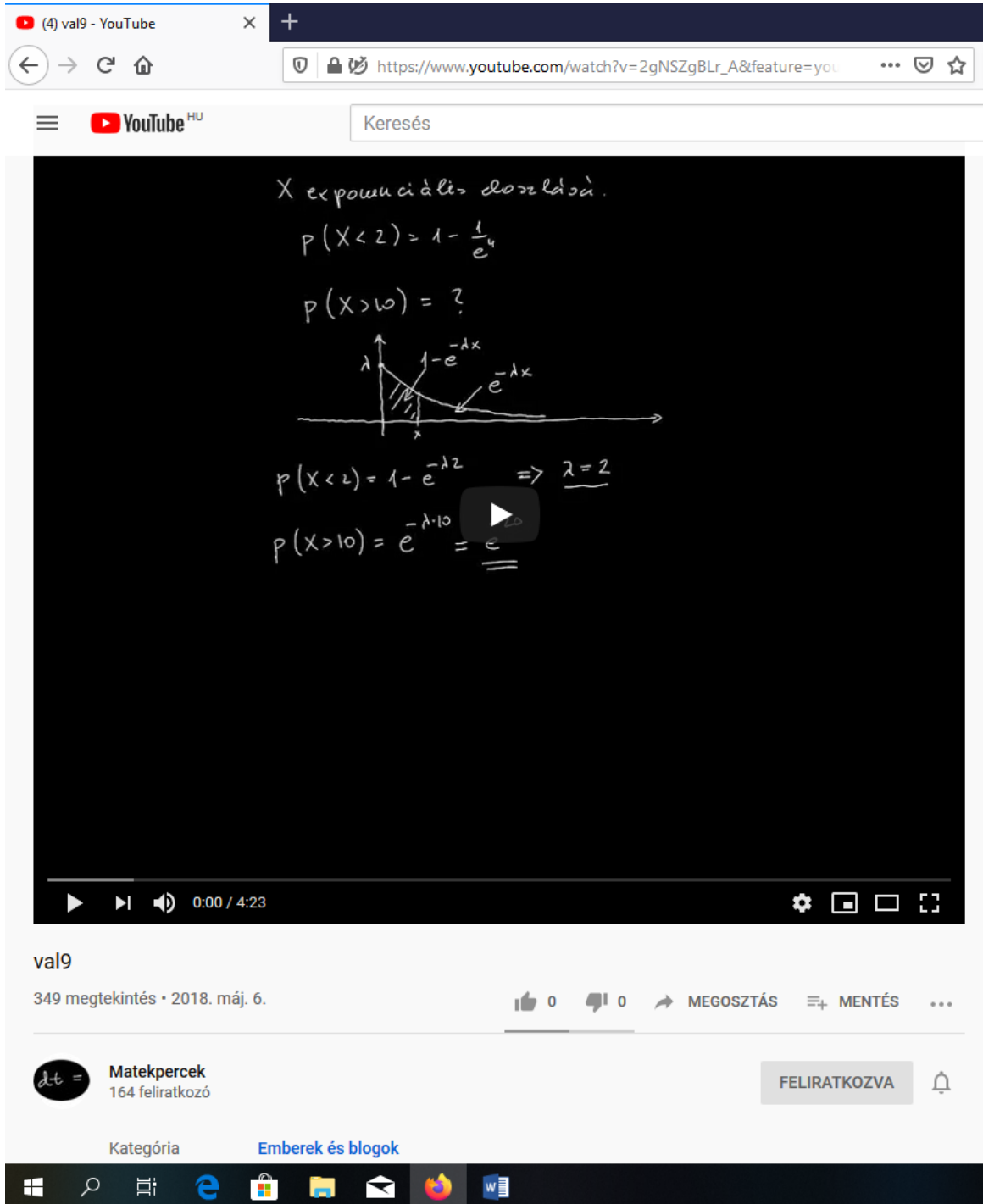
479 megtekintés • 2018. máj. 7.

Like: 3, Dislike: 0, Share: MEGOSZTÁS, Save: MENTÉS

Channel: Matekpercek (164 feliratkozó)

Subscription button: FELIRATKOZVA

Példa



The screenshot shows a YouTube video player with a handwritten mathematical solution. The text in the video is as follows:

$X$  exponenciális eloszlású.

$$P(X < 2) = 1 - \frac{1}{e^4}$$

$$P(X > 10) = ?$$

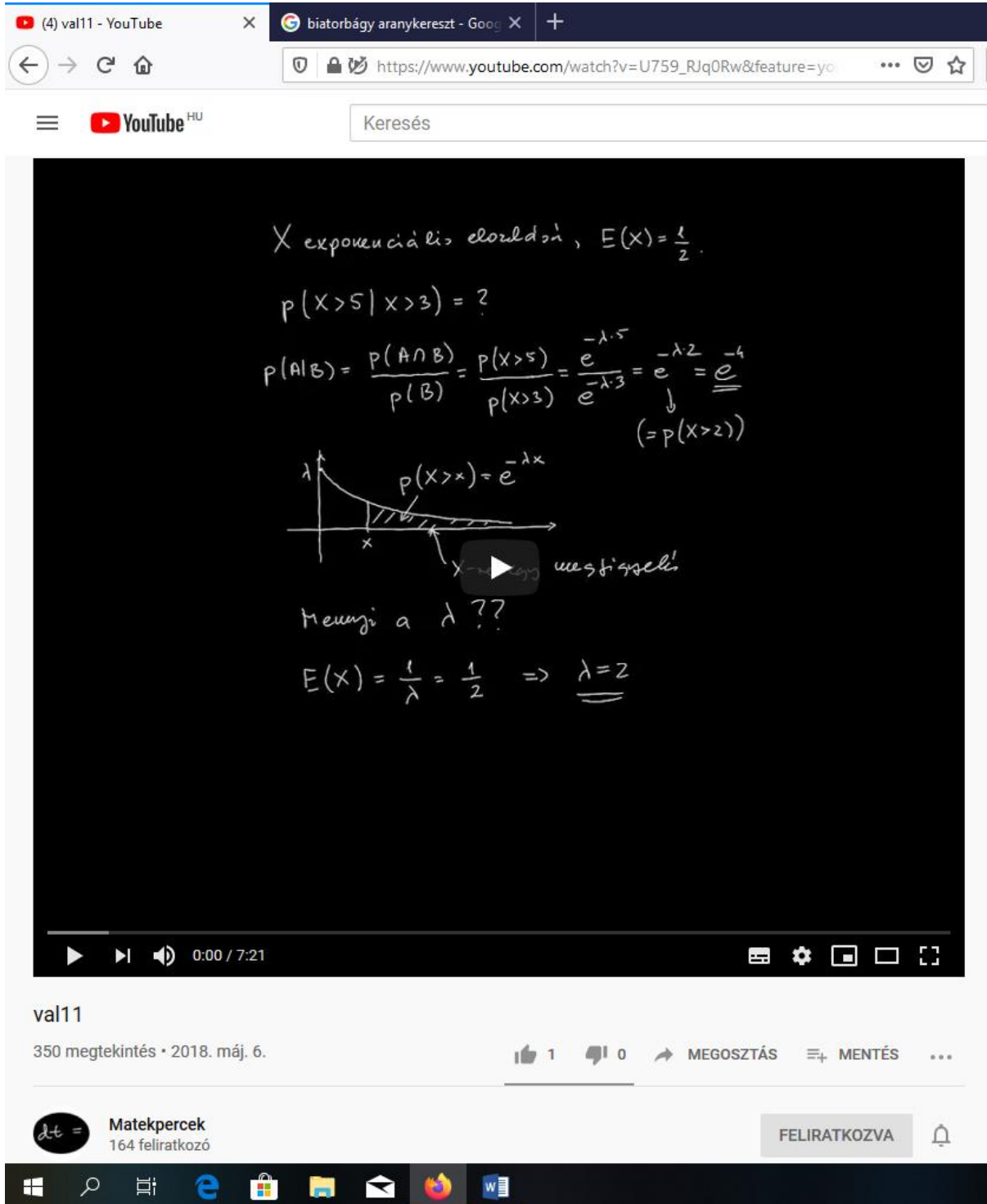
A graph of the exponential distribution function is shown, with the y-axis labeled  $\lambda$  and the x-axis labeled  $x$ . The curve is labeled  $1 - e^{-\lambda x}$  and  $e^{-\lambda x}$ . A vertical line is drawn at  $x = 2$ , and the area under the curve to the left of this line is shaded.

$$P(X < 2) = 1 - e^{-\lambda \cdot 2} \Rightarrow \underline{\lambda = 2}$$

$$P(X > 10) = e^{-\lambda \cdot 10} = \underline{\underline{e^{-20}}}$$

The video player interface shows the video title "val9", 349 views, and the date "2018. máj. 6.". The channel name is "Matekpercek" with 164 subscribers. The video is categorized under "Emberek és blogok".

Feltételes valószínűség



The video content includes the following text and equations:

$X$  exponenciális eloszlás,  $E(X) = \frac{1}{\lambda}$ .

$P(X > 5 | X > 3) = ?$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(X > 5)}{P(X > 3)} = \frac{e^{-\lambda \cdot 5}}{e^{-\lambda \cdot 3}} = e^{-\lambda \cdot 2} = e^{-4} = \underline{\underline{e^{-4}}}$$

( $= P(X > 2)$ )

A graph shows the exponential distribution curve  $p(x) = e^{-\lambda x}$  with a shaded area under the curve for  $x > 3$ . A note says "vesztéssel!" (with loss!).

Mennyi a  $\lambda$  ??

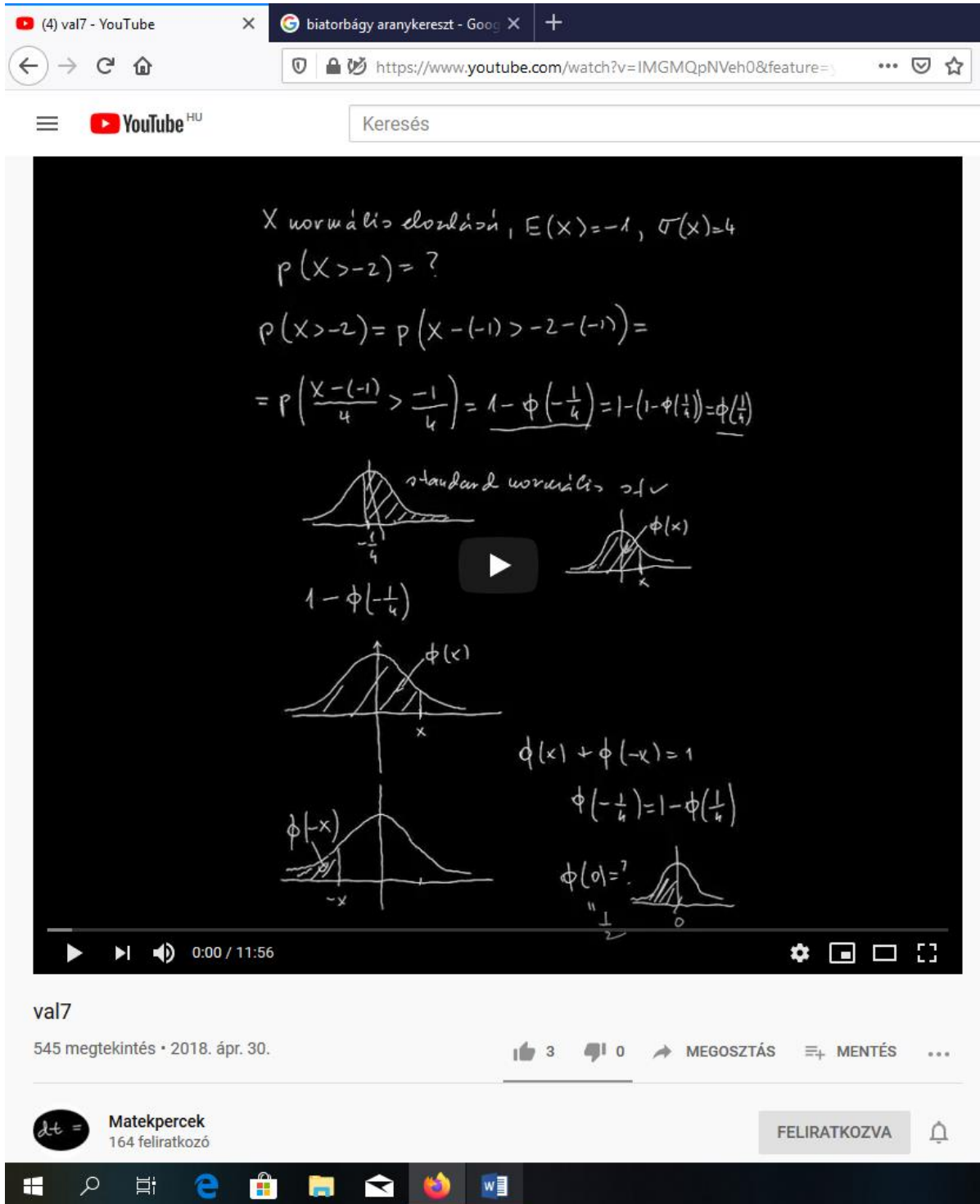
$$E(X) = \frac{1}{\lambda} = \frac{1}{2} \Rightarrow \underline{\underline{\lambda = 2}}$$

Video title: val11  
 350 megtekintés • 2018. máj. 6.  
 Like: 1, Dislike: 0, Share: MEGOSZTÁS, Save: MENTÉS  
 Channel: Matekpercek (164 feliratkozó), FELIRATKOZVA



Normális eloszás

Példa



(4) val7 - YouTube

biatorbágy aranykereszt - Google

https://www.youtube.com/watch?v=IMGMQpNVeh0&feature=...

Keresés

X normális eloszlású,  $E(X) = -1$ ,  $\sigma(X) = 4$   
 $P(X > -2) = ?$   
 $P(X > -2) = P(X - (-1) > -2 - (-1)) =$   
 $= P\left(\frac{X - (-1)}{4} > \frac{-1}{4}\right) = 1 - \Phi\left(-\frac{1}{4}\right) = 1 - (1 - \Phi\left(\frac{1}{4}\right)) = \Phi\left(\frac{1}{4}\right)$

standard normális  $\sigma=1$   
 $1 - \Phi\left(-\frac{1}{4}\right)$   
 $\Phi(x)$   
 $\Phi(x) + \Phi(-x) = 1$   
 $\Phi\left(-\frac{1}{4}\right) = 1 - \Phi\left(\frac{1}{4}\right)$   
 $\Phi(0) = ?$

val7

545 megtekintés • 2018. ápr. 30.

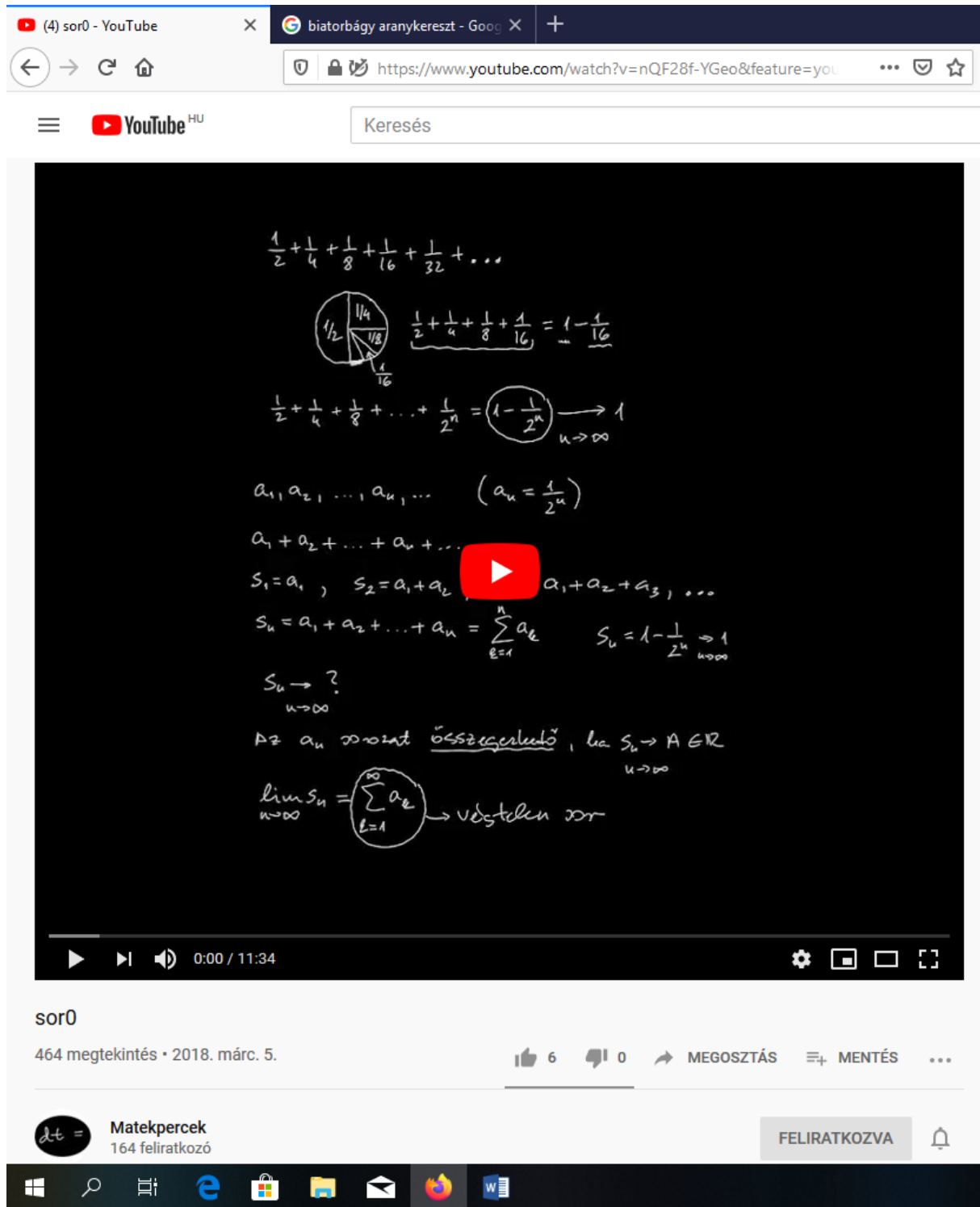
3 0 MEGOSZTÁS MENTÉS

Matekpercek  
164 feliratkozó


FELIRATKOZVA

## Sorok

A sor fogalma.  $1/2+1/4+1/8+$



$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots$


 $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 1 - \frac{1}{16}$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} = \left(1 - \frac{1}{2^n}\right) \xrightarrow{n \rightarrow \infty} 1$

$a_1, a_2, \dots, a_n, \dots \quad \left(a_n = \frac{1}{2^n}\right)$

$a_1 + a_2 + \dots + a_n + \dots$

$S_1 = a_1, \quad S_2 = a_1 + a_2, \quad a_1 + a_2 + a_3, \dots$

$S_n = a_1 + a_2 + \dots + a_n = \sum_{k=1}^n a_k \quad S_n = 1 - \frac{1}{2^n} \xrightarrow{n \rightarrow \infty} 1$

$S_n \rightarrow ?$   
 $n \rightarrow \infty$

Az  $a_n$  szigorúan összegeklődő, ha  $S_n \rightarrow A \in \mathbb{R}$   
 $n \rightarrow \infty$

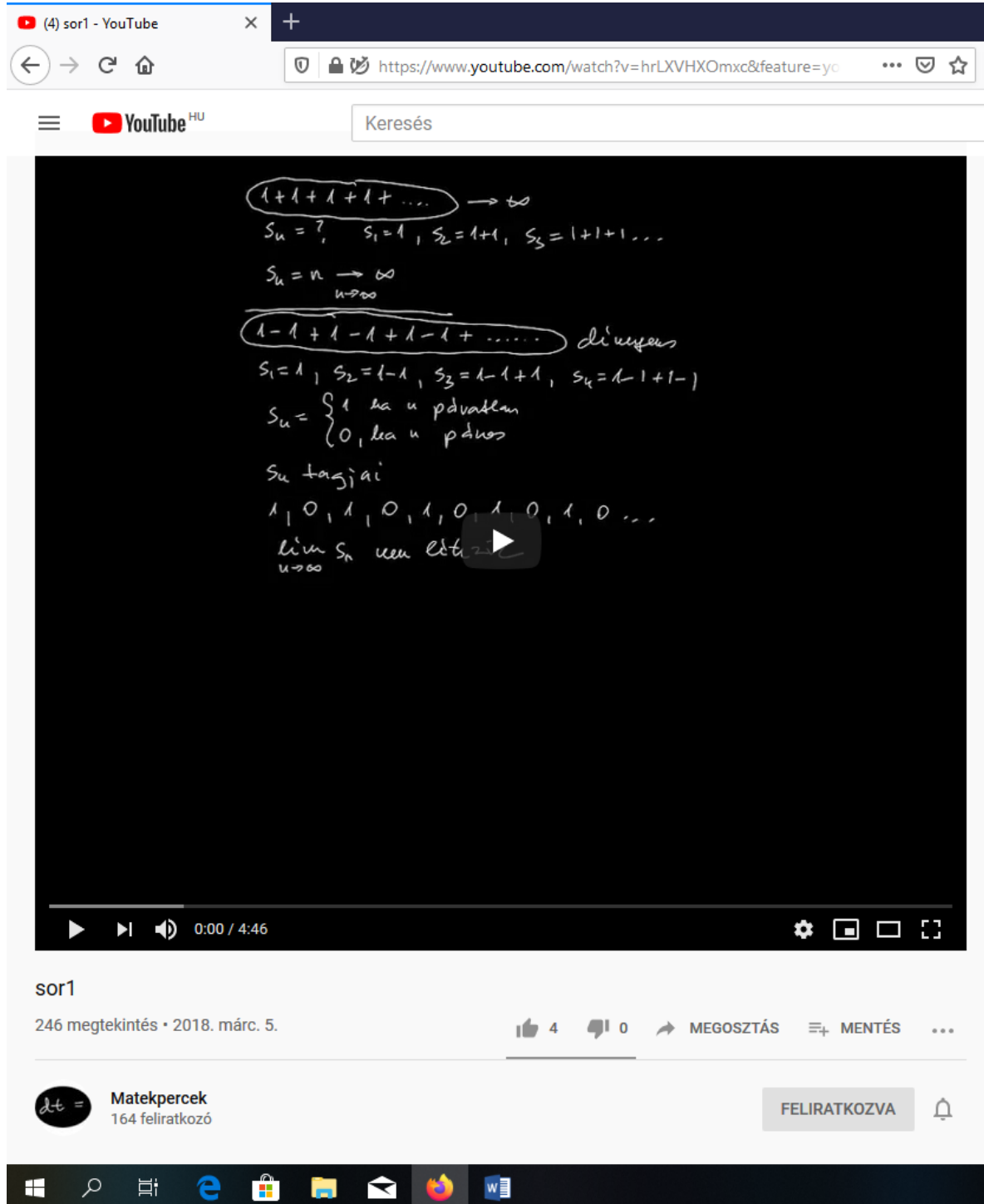
$\lim_{n \rightarrow \infty} S_n = \sum_{k=1}^{\infty} a_k \rightarrow$  végtelen sor

sor0  
 464 megtekintés · 2018. márc. 5.

Matekpercek  
 164 feliratkozó

FELIRATKOZVA

$1+1+1+1+\dots, 1-1+1-1+\dots$



Handwritten notes on a blackboard:

$1+1+1+1+\dots \rightarrow \infty$   
 $S_n = ?$ ,  $S_1 = 1$ ,  $S_2 = 1+1$ ,  $S_3 = 1+1+1, \dots$   
 $S_n = n \rightarrow \infty$   
 $n \rightarrow \infty$

$1-1+1-1+1-1+\dots$  *divergens*  
 $S_1 = 1$ ,  $S_2 = 1-1$ ,  $S_3 = 1-1+1$ ,  $S_4 = 1-1+1-1$   
 $S_n = \begin{cases} 1 & \text{ha } n \text{ páratlan} \\ 0 & \text{ha } n \text{ páros} \end{cases}$   
 $S_n$  tagjai  
 $1, 0, 1, 0, 1, 0, 1, 0, 1, 0, \dots$   
 $\lim_{n \rightarrow \infty} S_n$  nem létezik

Video player interface: (4) sor1 - YouTube, URL: https://www.youtube.com/watch?v=hrLXVHXOmx&feature=y...

Video title: sor1

246 megtekintés • 2018. márc. 5.

Like: 4, Comment: 0, Share: MEGOSZTÁS, Save: MENTÉS

Channel: Matekpercek (164 feliratkozó)

Subscribe: FELIRATKOZVA

konvergens  $\Rightarrow a_n \rightarrow 0$ . A harmonikus sor



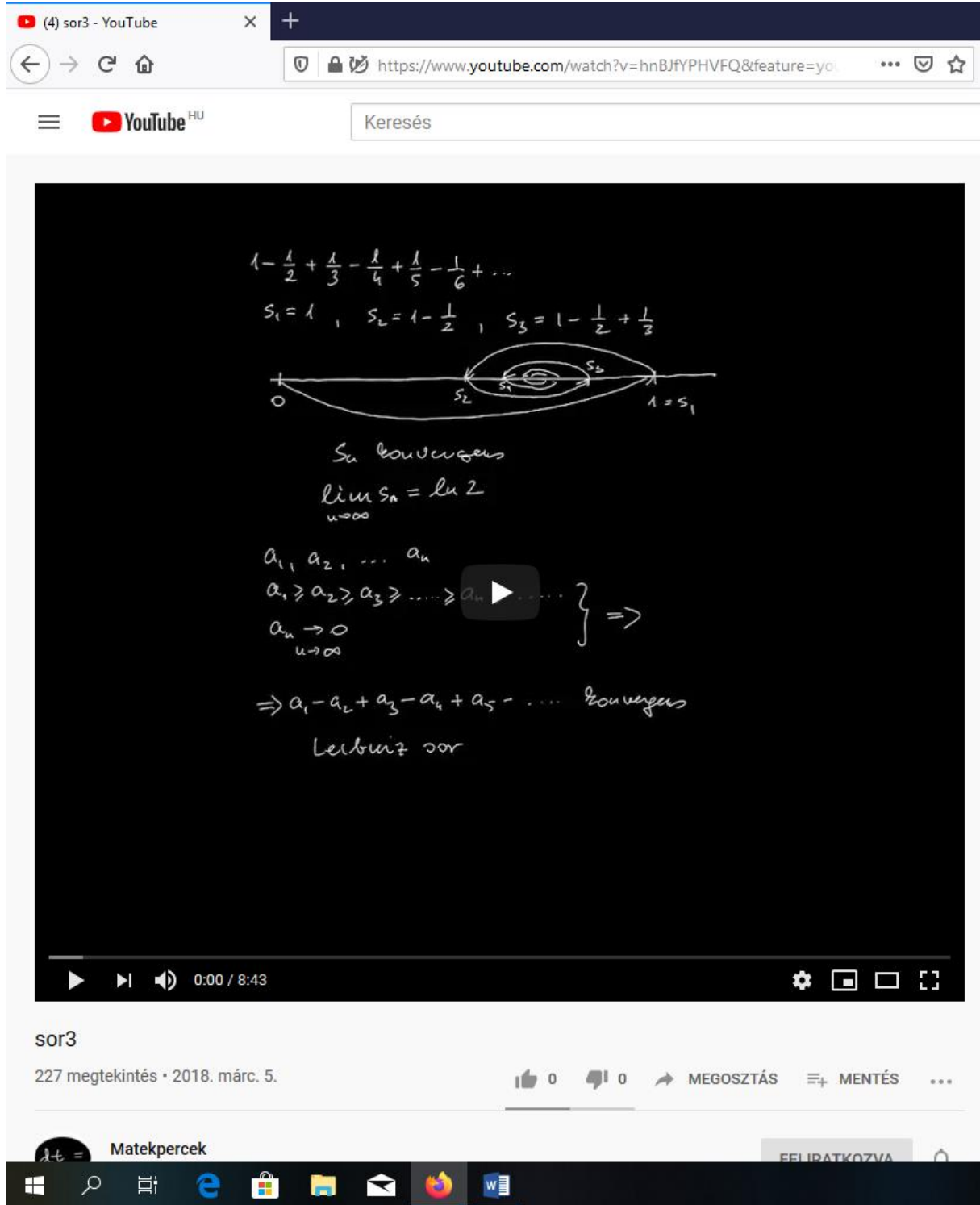
$a_1 + a_2 + \dots + a_n + \dots$  konvergens  $\Rightarrow a_n \rightarrow 0$   
 $1 - 1 + 1 - 1 + 1 - 1 + \dots$  divergens!  
 $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots + \frac{1}{2^n} + \dots$   
 $\rightarrow 0$   
 $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \frac{1}{15} + \frac{1}{16} + \frac{1}{17} + \frac{1}{18}$   
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$   
 $\frac{1}{2} \quad 4 \cdot \frac{1}{8} = \frac{1}{2} \quad 8 \cdot \frac{1}{16} = \frac{1}{2} \quad 0$   
 $2^{2000} = (2^4)^{500} = 16^{500} > 10^{700}$

sor2  
 286 megtekintés · 2018. márc. 5.

Matekpercek  
 164 feliratkozó

FELIRATKOZVA

Leibniz sorok.  $1 - 1/2 + 1/3 - 1/4 + 1/5 - 1/6 + \dots$



The video content includes the following handwritten text:

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots$$

$$s_1 = 1, \quad s_2 = 1 - \frac{1}{2}, \quad s_3 = 1 - \frac{1}{2} + \frac{1}{3}$$

A number line diagram shows points  $0, s_2, s_1, s_3, 1 = s_1$  with arcs indicating the partial sums.

$s_n$  konvergens  
 $\lim_{n \rightarrow \infty} s_n = \ln 2$

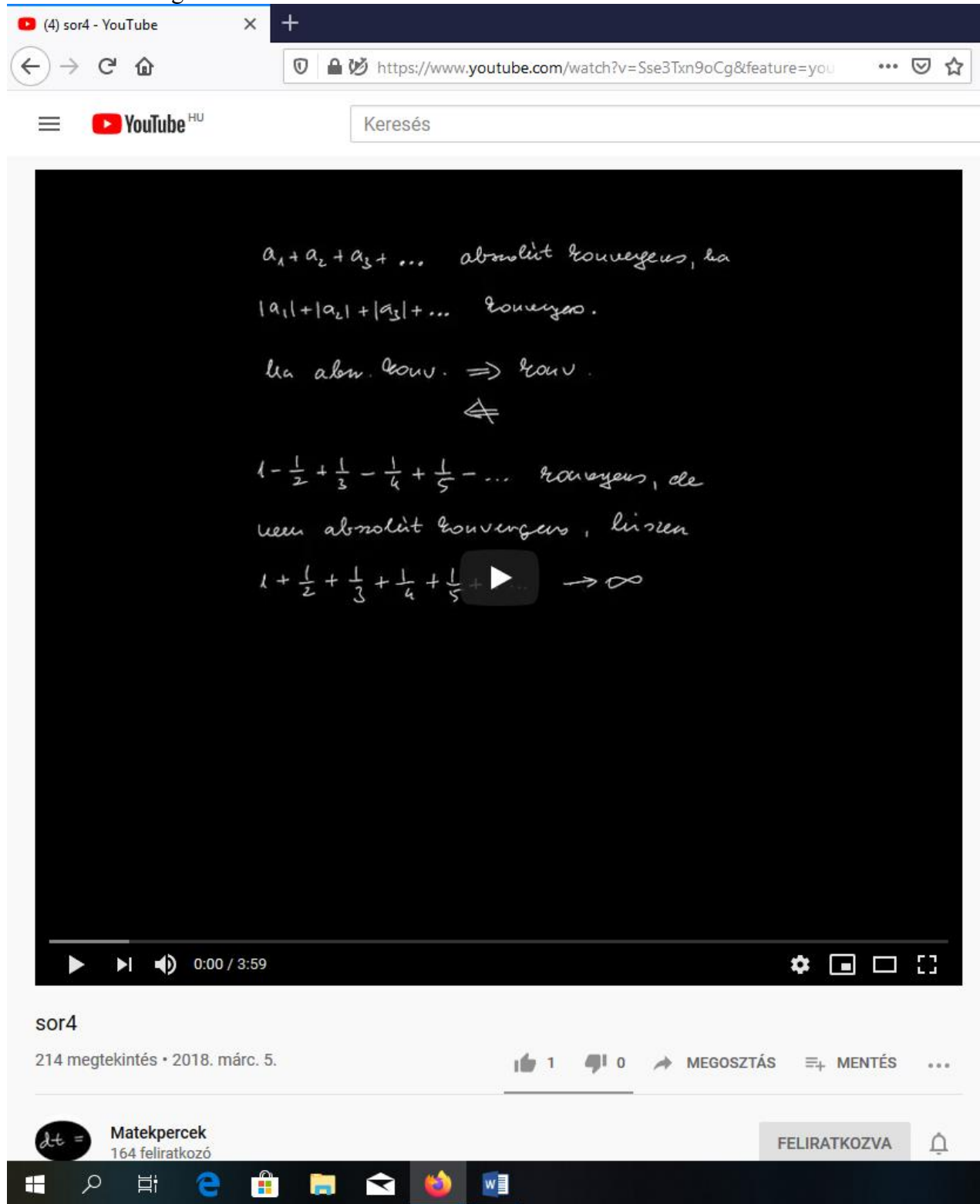
$a_1, a_2, \dots, a_n$   
 $a_1 \geq a_2 \geq a_3 \geq \dots \geq a_n \rightarrow 0$   
 $a_n \rightarrow 0$   
 $\Rightarrow a_1 - a_2 + a_3 - a_4 + a_5 - \dots$  konvergens  
 Leibniz sor

Video player controls: 0:00 / 8:43

Video title: sor3  
 227 megtekintés • 2018. márc. 5.

Channel: Matekpercek

Abszolút konvergencia.

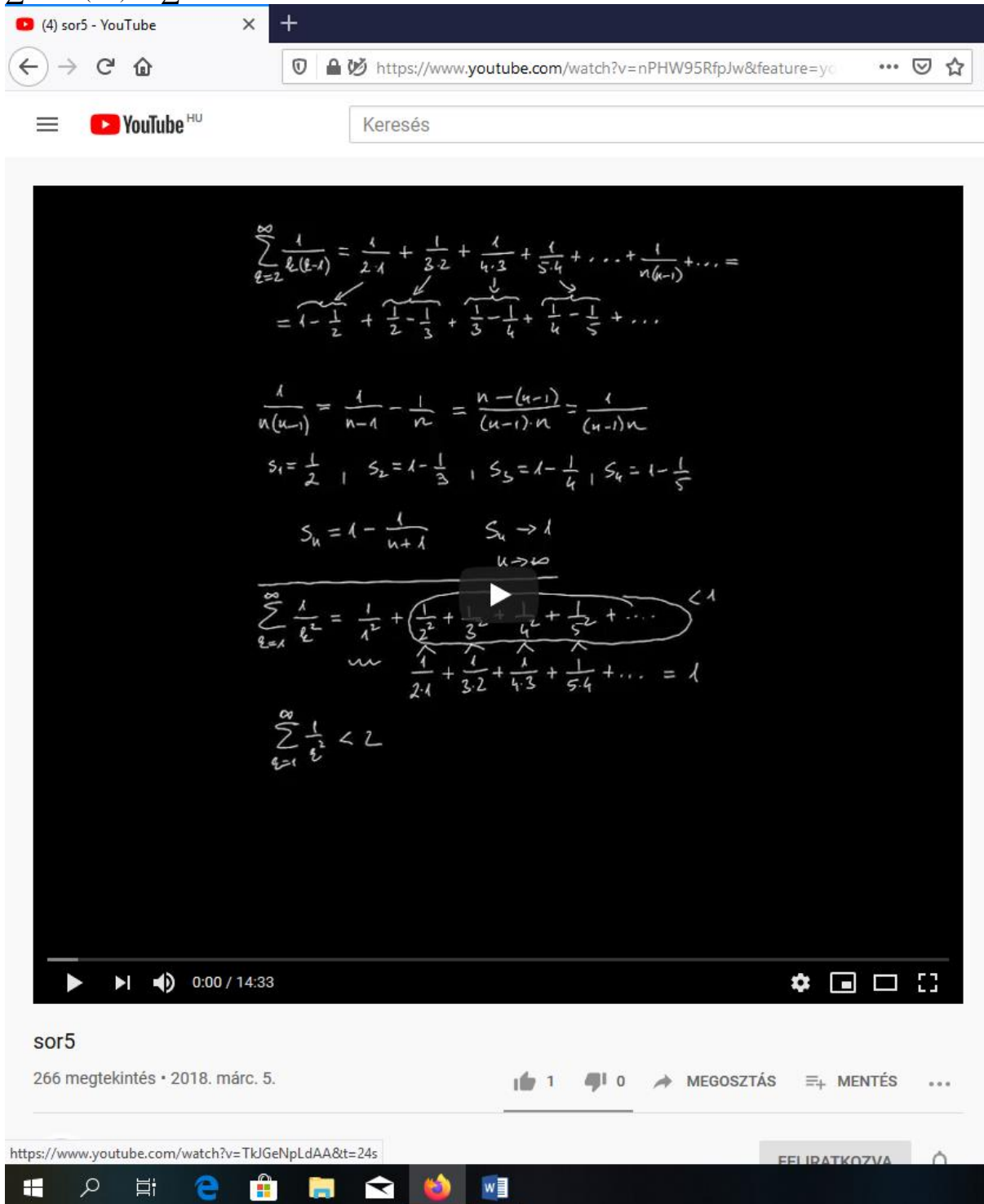


The screenshot shows a YouTube video player with the following content:

$a_1 + a_2 + a_3 + \dots$  abszolút konvergens, ha  
 $|a_1| + |a_2| + |a_3| + \dots$  konvergens.  
 ha absz. konv.  $\Rightarrow$  konv.  
 $\Leftarrow$   
 $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$  konvergens, de  
 nem abszolút konvergens, hiszen  
 $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots \rightarrow \infty$

Video title: sor4  
 Views: 214 megtekintés • 2018. márc. 5.  
 Likes: 1, Comments: 0  
 Channel: Matekpercek (164 feliratkozó)

$\sum_{k=2}^{\infty} \frac{1}{k(k-1)}$  és  $\sum_{k=1}^{\infty} \frac{1}{k^2}$



The video content includes the following mathematical derivations:

$$\sum_{k=2}^{\infty} \frac{1}{k(k-1)} = \frac{1}{2 \cdot 1} + \frac{1}{3 \cdot 2} + \frac{1}{4 \cdot 3} + \frac{1}{5 \cdot 4} + \dots + \frac{1}{n(n-1)} + \dots =$$

$$= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \dots$$

$$\frac{1}{n(n-1)} = \frac{1}{n-1} - \frac{1}{n} = \frac{n - (n-1)}{(n-1) \cdot n} = \frac{1}{(n-1)n}$$

$$S_1 = \frac{1}{2}, \quad S_2 = 1 - \frac{1}{3}, \quad S_3 = 1 - \frac{1}{4}, \quad S_4 = 1 - \frac{1}{5}$$

$$S_n = 1 - \frac{1}{n+1}, \quad S_n \rightarrow 1 \text{ as } n \rightarrow \infty$$


---

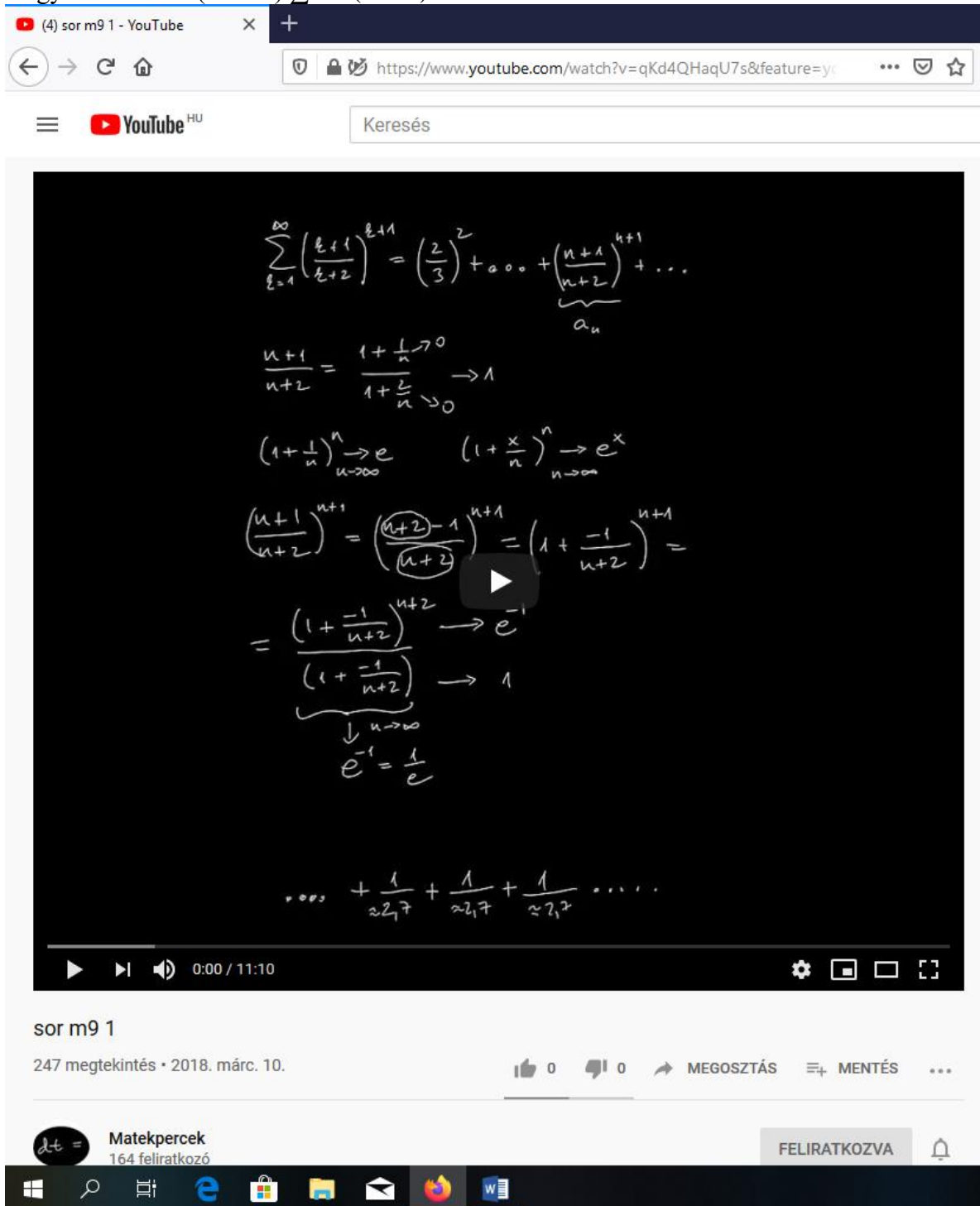

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{1}{1^2} + \left(\frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \dots\right) < 1$$

$$\frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \dots = \frac{1}{2 \cdot 1} + \frac{1}{3 \cdot 2} + \frac{1}{4 \cdot 3} + \frac{1}{5 \cdot 4} + \dots = 1$$

$$\sum_{k=1}^{\infty} \frac{1}{k^2} < 2$$

Video title: sor5  
 266 megtekintés • 2018. márc. 5.  
 1 like, 0 komment, MEGOSZTÁS, MENTÉS

Vegyes feladatok (kérdés)  $\sum_{k=1}^{\infty} (k+1k+2)k+1$



The video shows a handwritten mathematical derivation on a black background. The main equation is:

$$\sum_{k=1}^{\infty} \left( \frac{k+1}{k+2} \right)^{k+1} = \left( \frac{2}{3} \right)^2 + \dots + \underbrace{\left( \frac{n+1}{n+2} \right)^{n+1}}_{a_n} + \dots$$

The derivation then shows the limit of the term  $\frac{n+1}{n+2}$  as  $n \rightarrow \infty$ :

$$\frac{n+1}{n+2} = \frac{1 + \frac{1}{n} \rightarrow 0}{1 + \frac{2}{n} \rightarrow 0} \rightarrow 1$$

It also shows the limit of  $\left(1 + \frac{1}{n}\right)^n \rightarrow e$  and  $\left(1 + \frac{x}{n}\right)^n \rightarrow e^x$ .

Then, it rewrites the term  $\left(\frac{n+1}{n+2}\right)^{n+1}$  as:

$$\left(\frac{n+1}{n+2}\right)^{n+1} = \left(\frac{(n+2)-1}{n+2}\right)^{n+1} = \left(1 + \frac{-1}{n+2}\right)^{n+1}$$

$$= \frac{\left(1 + \frac{-1}{n+2}\right)^{n+2}}{\left(1 + \frac{-1}{n+2}\right)} \rightarrow \frac{e^{-1}}{1} = e^{-1} = \frac{1}{e}$$

At the bottom, it shows the series terms:  $\dots + \frac{1}{\approx 2,7} + \frac{1}{\approx 2,7} + \frac{1}{\approx 2,7} \dots$

Video title: sor m9 1  
 247 megtekintés • 2018. márc. 10.  
 Matekpercek (164 feliratkozó)  
 FELIRATKOZVA



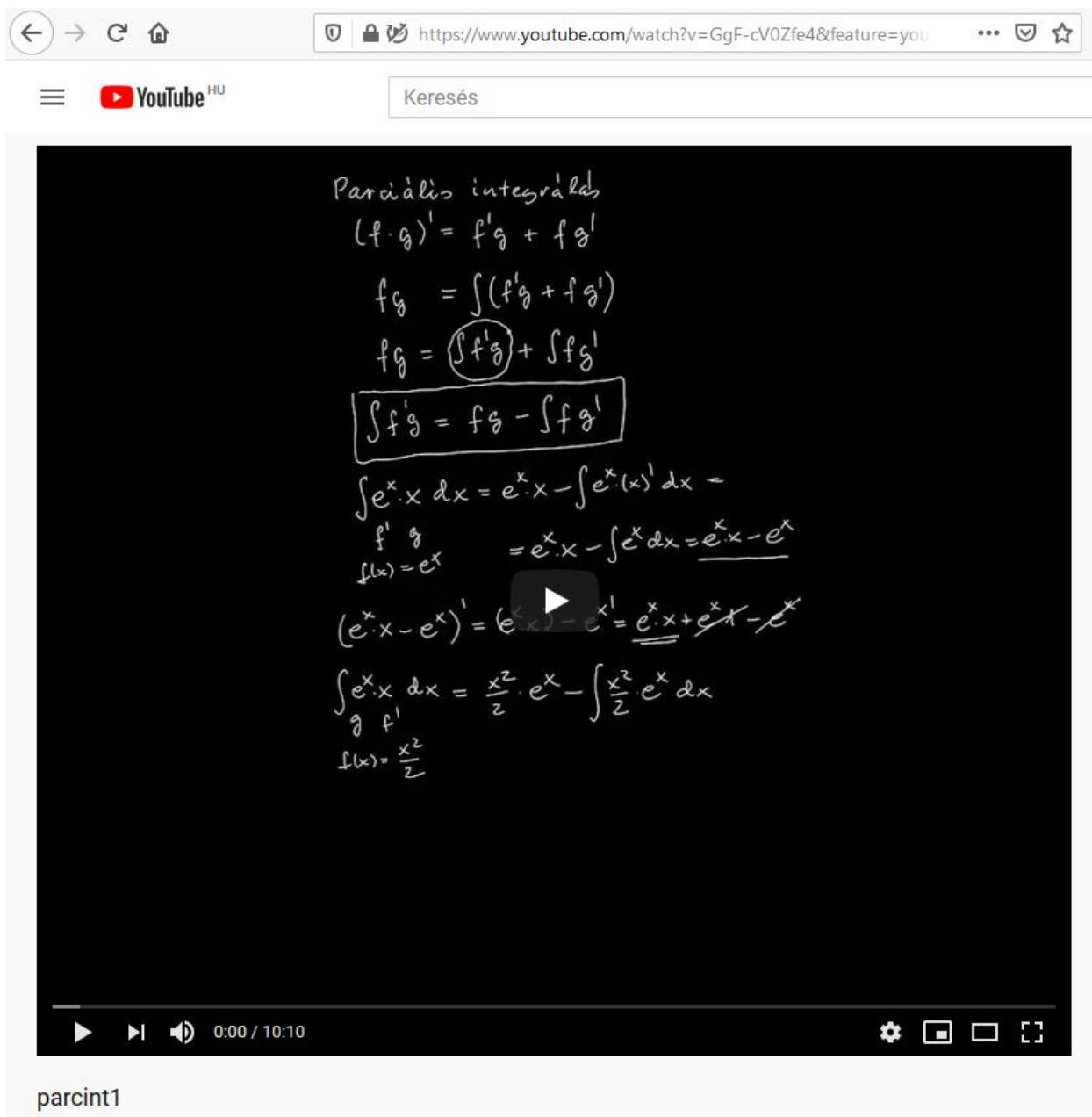
## Integrálok

Határozott integrál

Határozatlan integrálok

Parciális integrálok

Bevezetés.1.  $\int e^x dx$



Parciális integrálás

$$(f \cdot g)' = f'g + fg'$$

$$fg = \int (f'g + fg')$$

$$fg = \int f'g + \int fg'$$

$$\int f'g = fg - \int fg'$$

$$\int e^x \cdot x dx = e^x \cdot x - \int e^x (x)' dx -$$

$$\int f'g = e^x \cdot x - \int e^x dx = e^x \cdot x - e^x$$

$$(e^x \cdot x - e^x)' = (e^x)' \cdot x - e^x = e^x \cdot x + e^x - e^x$$

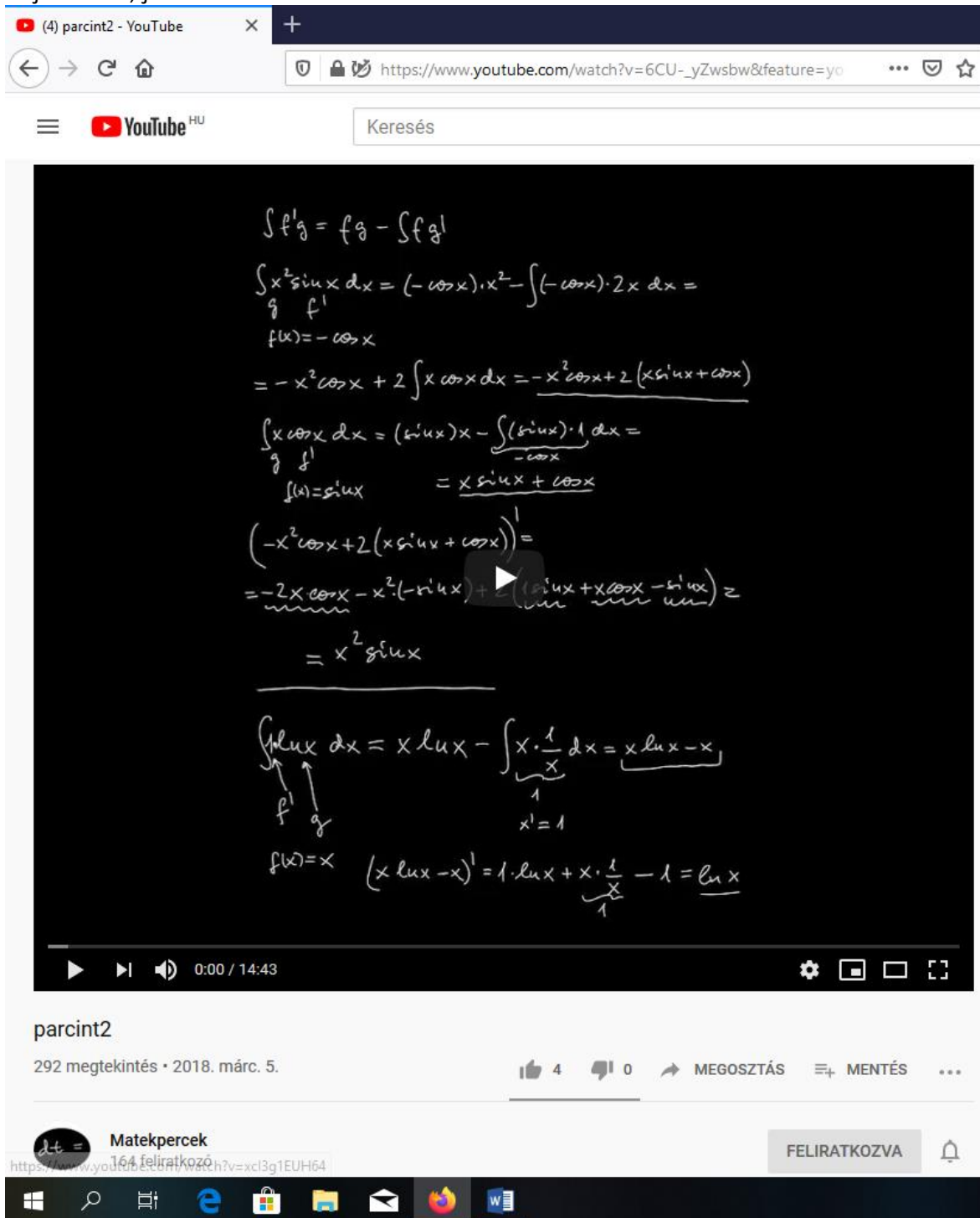
$$\int e^x \cdot x dx = \frac{x^2}{2} \cdot e^x - \int \frac{x^2}{2} e^x dx$$

$$f(x) = \frac{x^2}{2}$$

0:00 / 10:10

parcint1

2.  $\int x^2 \sin x \, dx$ ,  $\int \ln x \, dx$



The video content shows the following mathematical work:

**Integration of  $\int x^2 \sin x \, dx$ :**

$$\int f'g = fg - \int fg'$$

$$\int x^2 \sin x \, dx = (-\cos x) \cdot x^2 - \int (-\cos x) \cdot 2x \, dx =$$

$$f(x) = -\cos x$$

$$= -x^2 \cos x + 2 \int x \cos x \, dx = -x^2 \cos x + 2(x \sin x + \cos x)$$

$$\int x \cos x \, dx = (\sin x)x - \int (\sin x) \cdot 1 \, dx =$$

$$f(x) = \sin x \quad = x \sin x + \cos x$$

$$\left(-x^2 \cos x + 2(x \sin x + \cos x)\right)' =$$

$$= -2x \cos x - x^2(-\sin x) + 2(\sin x + x \cos x - \sin x) =$$

$$= x^2 \sin x$$

**Integration of  $\int \ln x \, dx$ :**

$$\int f'g \, dx = x \ln x - \int x \cdot \frac{1}{x} \, dx = x \ln x - x$$

$$f(x) = x \quad (x \ln x - x)' = 1 \cdot \ln x + x \cdot \frac{1}{x} - 1 = \ln x$$

Video player controls: 0:00 / 14:43

parcint2  
292 megtekintés • 2018. márc. 5.

Matekpercek  
164 feliratkozó

FELIRATKOZVA

3.  $\int e^x \sin x \, dx$



The video shows the following steps for the integration of  $\int e^x \sin x \, dx$ :

$$\int f'g = fg - \int fg'$$

$$\int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx$$

where  $f(x) = e^x$ .

$$\int e^x \cos x \, dx = e^x \cos x - \int e^x (-\sin x) \, dx = e^x \cos x + \int e^x \sin x \, dx$$

$$\int e^x \sin x \, dx = e^x \sin x - (e^x \cos x + \int e^x \sin x \, dx)$$

$$\int e^x \sin x \, dx = e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$$

$$2 \int e^x \sin x \, dx = e^x \sin x - e^x \cos x$$

$$\int e^x \sin x \, dx = \frac{1}{2} (e^x \sin x - e^x \cos x)$$

$$\frac{1}{2} (e^x \sin x - e^x \cos x)' =$$

$$= \frac{1}{2} (e^x \sin x + e^x \cos x - e^x \cos x - e^x (-\sin x)) =$$

$$= \frac{1}{2} (2e^x \sin x) = e^x \sin x$$

The video title is "parcint3" and it has 261 views as of March 5, 2018. The channel is "Matekpercek" with 164 subscribers.

## Helyettesítéses integrálok

1. Bevezetés.  $\int e^{\sin x} \cos x \, dx$ ,  $\int x \sin(x^2) \, dx$



The screenshot shows a YouTube video player with handwritten mathematical derivations for substitution integration. The video title is "helyint1" and it has 382 views as of March 5, 2018.

The derivations shown in the video are:

$$F(g(x))' = F'(g(x)) \cdot g'(x)$$

$$F(g(x)) = \int F'(g(x)) \cdot g'(x) \quad f := F'$$

$$\int f(g(x)) \cdot g'(x) dx = F(g(x)) \quad F = \int f$$

$$t = g(x) \quad dt = g'(x) dx$$

$$\int f(t) \cdot dt = F(t) = F(g(x))$$

Example 1:  $\int e^{\sin x} \cos x \, dx$

$$\int e^{\sin x} \cos x \, dx = \int e^t dt = e^t = e^{\sin x}$$

$$t = \sin x \quad dt = \cos x \, dx$$

Example 2:  $\int e^{\sin(x^2)} \cdot g'(x) dx = \int e^{g(x)} \cdot g'(x) dx$

$$\int \sin(x^2) \cdot x \, dx = \frac{1}{2} \int \sin(x^2) \cdot 2x \, dx =$$

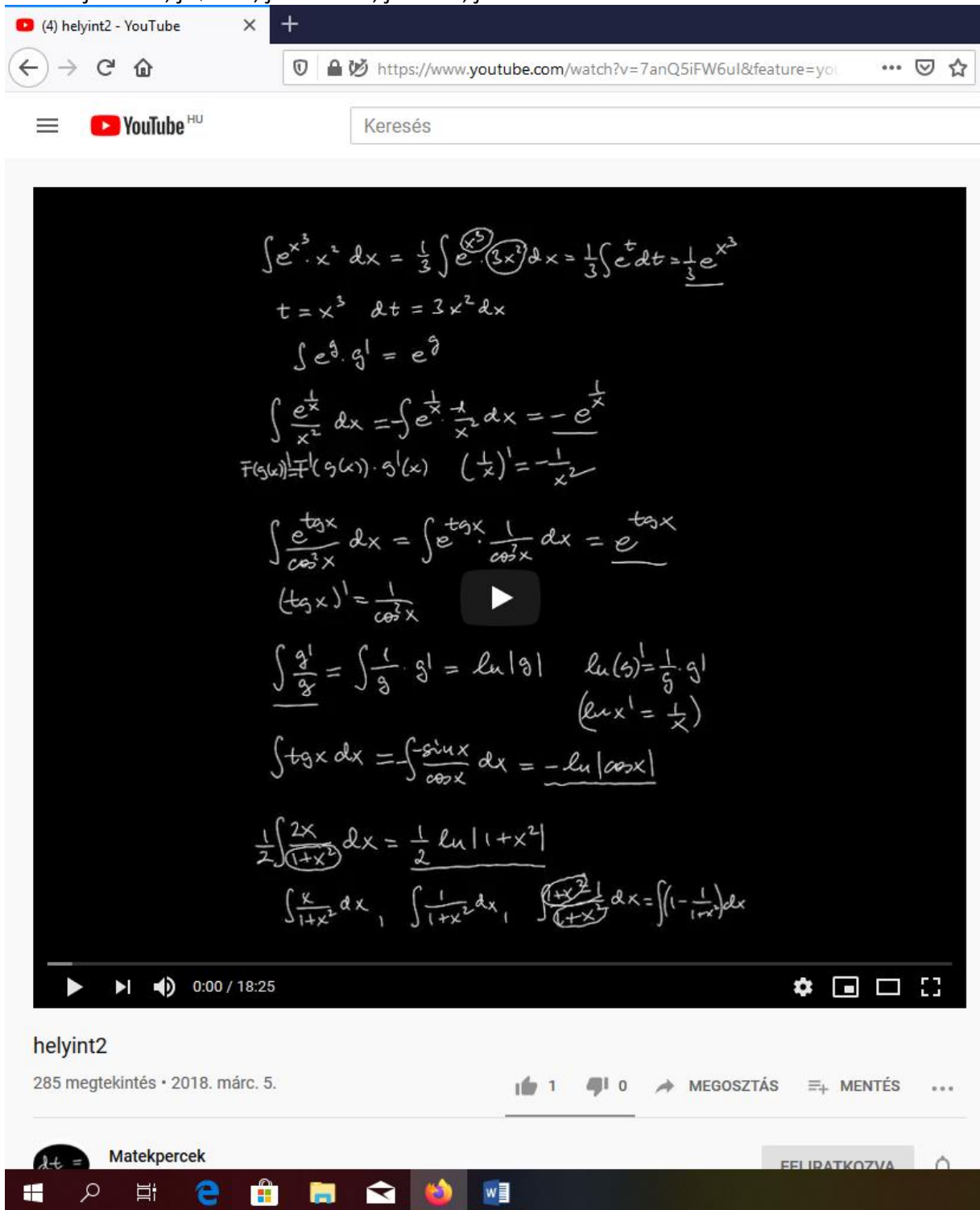
$$= \frac{1}{2} \int \sin(t) dt = -\frac{1}{2} \cos(t) = -\frac{1}{2} \cos(x^2)$$

$$t = x^2 \quad dt = 2x \, dx$$

$$\left(-\frac{1}{2} \cos(x^2)\right)' = -\frac{1}{2} (\cos(x^2))' = -\frac{1}{2} (-\sin(x^2)) \cdot 2x =$$

$$= \underline{\underline{\sin(x^2) \cdot x}}$$

2.  $\int e^{x^3} x^2 dx$ ,  $\int e^{1/x} x^2 dx$ ,  $\int e^{\tan x} \cos^2 x dx$ ,  $\int \tan x dx$ ,  $\int x^{1+x^2} dx$



The screenshot shows a YouTube video player with a black background and white handwritten text. The video content includes the following mathematical work:

$$\int e^{x^3} \cdot x^2 dx = \frac{1}{3} \int e^{t} \cdot 3x^2 dx = \frac{1}{3} \int e^t dt = \frac{1}{3} e^{x^3}$$

$$t = x^3 \quad dt = 3x^2 dx$$

$$\int e^g \cdot g' = e^g$$

$$\int \frac{e^{\frac{1}{x}}}{x^2} dx = \int e^{\frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right) dx = -e^{\frac{1}{x}}$$

$$F(g(x)) = F(g(x)) \cdot g'(x) \quad \left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$\int \frac{e^{\tan x}}{\cos^2 x} dx = \int e^{\tan x} \cdot \frac{1}{\cos^2 x} dx = e^{\tan x}$$

$$(\tan x)' = \frac{1}{\cos^2 x}$$

$$\int \frac{g'}{g} = \int \frac{1}{g} \cdot g' = \ln|g| \quad \ln(g)' = \frac{1}{g} \cdot g'$$

$$(\ln x)' = \frac{1}{x}$$

$$\int \tan x dx = \int \frac{-\sin x}{\cos x} dx = -\ln|\cos x|$$

$$\frac{1}{2} \int \frac{2x}{1+x^2} dx = \frac{1}{2} \ln|1+x^2|$$

$$\int \frac{x}{1+x^2} dx, \int \frac{1}{1+x^2} dx, \int \frac{1+x^2}{1+x^2} dx = \int \left(1 - \frac{1}{1+x^2}\right) dx$$

Below the video player, the channel name "helyint2" is visible, along with the view count "285 megtekintés" and the date "2018. márc. 5.". The video player controls show a play button and a progress bar at 0:00 / 18:25.

3.  $\int 1/x \ln x \, dx$ ,  $\int \sin^3 x \cos x \, dx$ ,  $\int \ln^2 x \, dx$ ,  $\int e^x / (1+e^{2x}) \, dx$

The video content shows the following solutions:

$$\int \frac{1}{x \cdot \ln x} dx = \int \frac{1/x}{\ln x} dx = \ln |\ln x|$$

$$\int \frac{g'}{g} = \ln |g|$$

$$\int g^3 \cdot g' = \frac{1}{4} g^4 \quad \int g^\alpha \cdot g' = \frac{g^{\alpha+1}}{\alpha+1} \quad (\alpha \neq -1)$$

$$x^4 = 4x^3 \quad g^4 = 4g^3 \cdot g'$$

$$\int \sin^3 x \cdot \cos x \, dx = \frac{1}{4} \sin^4 x$$

Substitution:  $t = \sin x \quad dt = \cos x \, dx$

$$\int t^3 \, dt = \frac{t^4}{4} = \frac{\sin^4 x}{4}$$

$$\int \frac{\ln^2 x}{x} dx = \int \ln^2 x \cdot \frac{1}{x} dx = \frac{\ln^3 x}{3}$$

$$\left(\frac{1}{3} \ln^3 x\right)' = \frac{1}{3} \cdot 3 \cdot (\ln x)^2 \cdot \frac{1}{x}$$

$$\int \frac{e^x}{1+e^{2x}} dx = \int \frac{1}{1+(e^x)^2} (e^x)' dx = \arctan e^x$$

Substitution:  $t = e^x \quad dt = e^x dx$

$$\arctan x' = \frac{1}{1+x^2} \quad \int \frac{1}{1+t^2} dt = \arctan t$$

Video player controls: 0:00 / 14:56

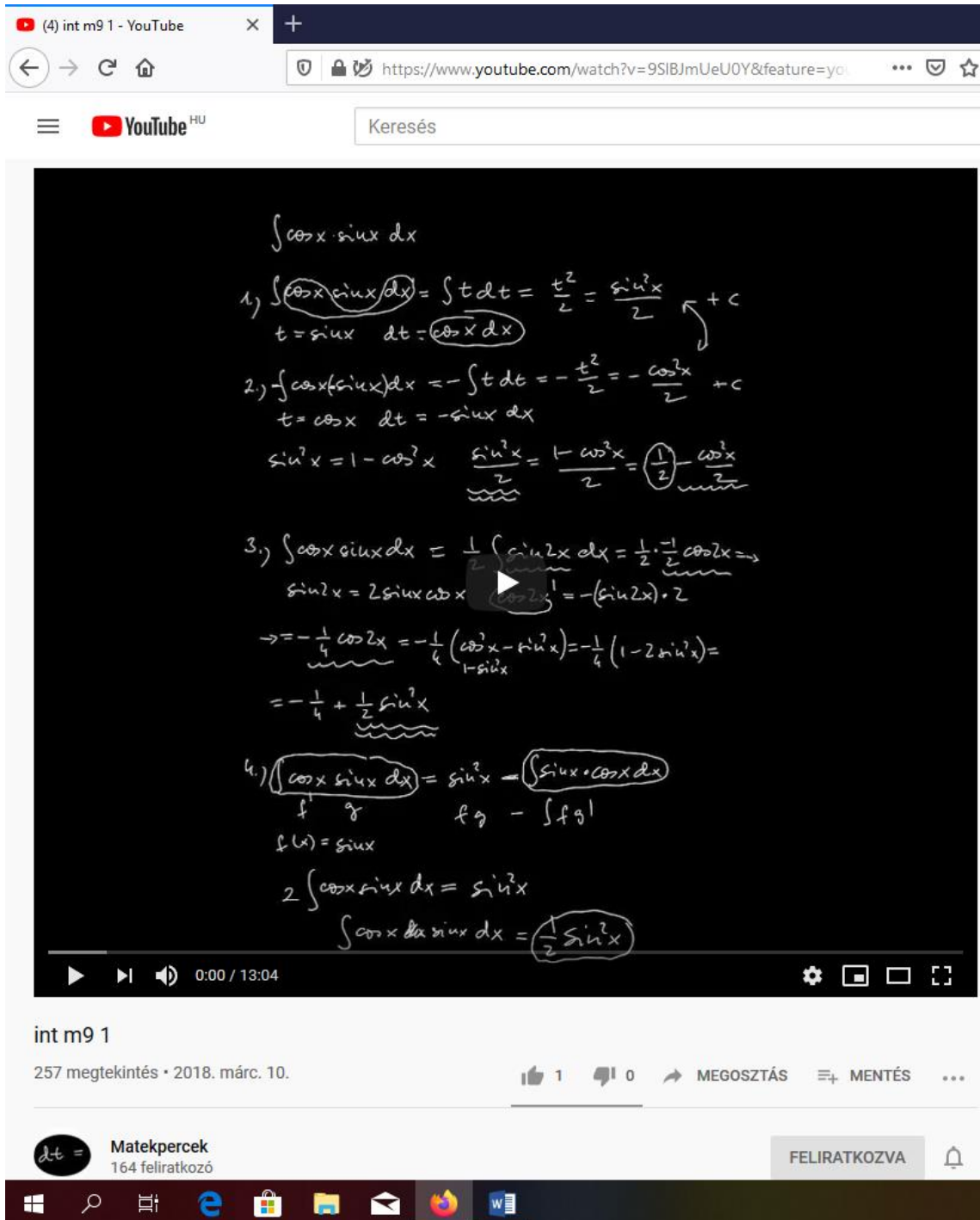
helyint3  
245 megtekintés • 2018. márc. 5.

Matekpercek  
164 feliratkozó

FELIRATKOZVA

## Improprius integrálok Vegyes feladatok

### 1. (kérdés) $\int \cos x \sin x \, dx$



The screenshot shows a YouTube video player with handwritten mathematical work on a black background. The work is organized into four numbered steps:

- $$\int \cos x \cdot \sin x \, dx$$

$$1.) \int \cos x \sin x \, dx = \int t \, dt = \frac{t^2}{2} = \frac{\sin^2 x}{2} + C$$

$t = \sin x \quad dt = \cos x \, dx$
- $$2.) \int \cos x \sin x \, dx = -\int t \, dt = -\frac{t^2}{2} = -\frac{\cos^2 x}{2} + C$$

$t = \cos x \quad dt = -\sin x \, dx$

$$\sin^2 x = 1 - \cos^2 x \quad \frac{\sin^2 x}{2} = \frac{1 - \cos^2 x}{2} = \frac{1}{2} - \frac{\cos^2 x}{2}$$
- $$3.) \int \cos x \sin x \, dx = \frac{1}{2} \int \sin 2x \, dx = \frac{1}{2} \cdot \frac{-1}{2} \cos 2x = -\frac{1}{4} \cos 2x$$

$$\sin 2x = 2 \sin x \cos x \quad \cos 2x = -(\sin 2x) \cdot 2$$

$$\rightarrow -\frac{1}{4} \cos 2x = -\frac{1}{4} \frac{\cos^2 x - \sin^2 x}{1 - \sin^2 x} = -\frac{1}{4} (1 - 2 \sin^2 x) = -\frac{1}{4} + \frac{1}{2} \sin^2 x$$
- $$4.) \int \cos x \sin^4 x \, dx = \sin^4 x - \int \sin^3 x \cos x \, dx$$

$f' \cdot g \quad f \cdot g' - \int f \cdot g'$

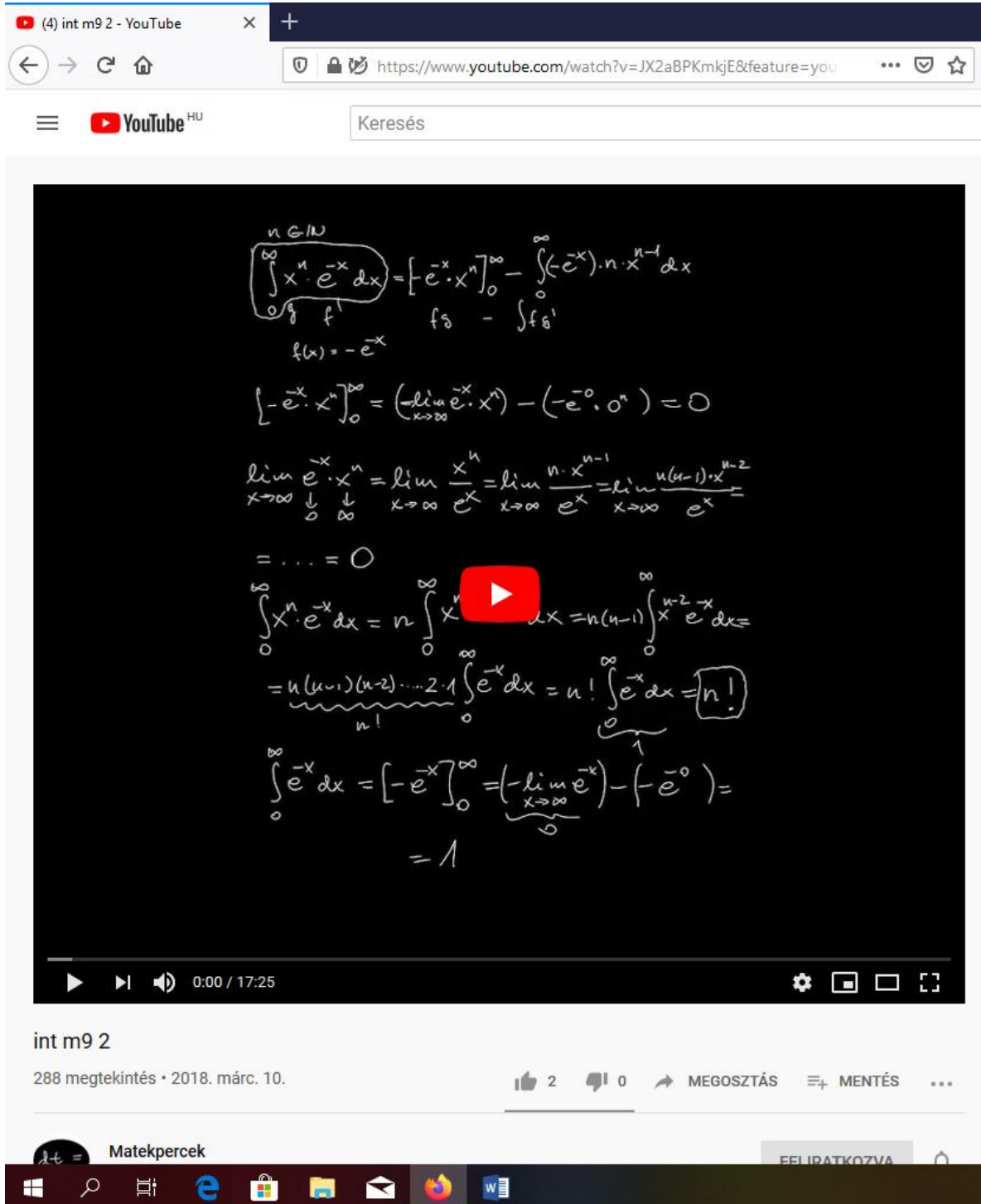
$f(x) = \sin^4 x$

$$2 \int \cos x \sin^4 x \, dx = \sin^4 x$$

$$\int \cos x \sin^4 x \, dx = \frac{1}{2} \sin^4 x$$

Below the video player, the video title is "int m9 1", the view count is "257 metekintés", and the date is "2018. márc. 10.". The channel name is "Matekpercek" with 164 subscribers. The video is marked as "FELIRATKOZVA".

2. (kérdés)  $\int_0^{\infty} x^n e^{-x} dx$



The video shows a handwritten derivation for the integral  $\int_0^{\infty} x^n e^{-x} dx$  where  $n \in \mathbb{N}$ . The derivation uses integration by parts and a limit process to show that the integral equals  $n!$ .

$$\int_0^{\infty} x^n e^{-x} dx = \left[ -e^{-x} x^n \right]_0^{\infty} - \int_0^{\infty} (-e^{-x}) \cdot n x^{n-1} dx$$

$$f(x) = -e^{-x}$$

$$\left[ -e^{-x} x^n \right]_0^{\infty} = \left( \lim_{x \rightarrow \infty} -e^{-x} x^n \right) - \left( -e^{-0} \cdot 0^n \right) = 0$$

$$\lim_{x \rightarrow \infty} \frac{-e^{-x} x^n}{\infty} = \lim_{x \rightarrow \infty} \frac{-x^n}{e^x} = \lim_{x \rightarrow \infty} \frac{-n x^{n-1}}{e^x} = \lim_{x \rightarrow \infty} \frac{-n(n-1) x^{n-2}}{e^x} = \dots = 0$$

$$\int_0^{\infty} x^n e^{-x} dx = n \int_0^{\infty} x^{n-1} e^{-x} dx = n(n-1) \int_0^{\infty} x^{n-2} e^{-x} dx = \dots = n(n-1)(n-2) \dots 2 \cdot 1 \int_0^{\infty} e^{-x} dx = n! \int_0^{\infty} e^{-x} dx = n! \cdot 1 = n!$$

$$\int_0^{\infty} e^{-x} dx = \left[ -e^{-x} \right]_0^{\infty} = \left( \lim_{x \rightarrow \infty} -e^{-x} \right) - \left( -e^{-0} \right) = 1$$

Video title: int m9 2  
 Views: 288 megtekintés • 2018. márc. 10.  
 Likes: 2, Comments: 0, Share: MEGOSZTÁS, Save: MENTÉS

Channel: Matekpercek



3. (kérdés)  $\int_{-\infty}^{\infty} 3xe^{-3x} dx$

Handwritten mathematical solution for the integral  $\int_{-\infty}^{\infty} 3xe^{-3x} dx$ :

$$\int_0^{\infty} 3xe^{-3x} dx = \left[ \frac{1}{3} e^{-3x} \cdot 3 \right]_0^{\infty} - \int_0^{\infty} \left( -\frac{1}{3} e^{-3x} \right) \cdot 3 dx =$$

$$fg - \int f'g'$$

$$f(x) = -\frac{1}{3} e^{-3x}$$

$$e^{-3x}' = e^{-3x} \cdot (-3)$$

$$= \left[ -e^{-3x} \cdot x \right]_0^{\infty} + \int_0^{\infty} e^{-3x} dx = \left[ -\frac{1}{3} e^{-3x} \right]_0^{\infty} = 0 - \left( -\frac{1}{3} \right) = \frac{1}{3}$$

$$\left[ e^{-3x} \cdot x \right]_0^{\infty} = \left( \lim_{x \rightarrow \infty} e^{-3x} \cdot x \right) - \left( e^{-3 \cdot 0} \cdot 0 \right) = 0$$

$$\lim_{x \rightarrow \infty} e^{-3x} \cdot x = \lim_{x \rightarrow \infty} \frac{x}{e^{3x}} = \lim_{x \rightarrow \infty} \frac{1}{e^3} = 0$$

Video player controls: 0:00 / 9:44

int m9 3  
 227 megtekintés • 2018. márc. 10.  
 1 like, 0 komment, MEGOSZTÁS, MENTÉS

Matekpercek  
 164 feliratkozó  
 FELIRATKOZVA